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The effect of observables, functional specifications, model features and shocks on identification in linearized DSGE models☆

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ABSTRACT

The decisions a researcher makes at the model building stage are crucial for parameter identification. This paper contains a number of applied tips for solving identifiability problems and improving the strength of DSGE model parameter identification by fine-tuning the (1) choice of observables, (2) functional specifications, (3) model features and (4) choice of structural shocks. We offer a formal approach based on well-established diagnostics and indicators to uncover and address both theoretical (yes/no) identifiability issues and weak identification from a Bayesian perspective. The concepts are illustrated by two exemplary models that demonstrate the identification properties of different investment adjustment cost specifications and output-gap definitions. Our results provide theoretical support for the use of growth adjustment costs, investment-specific technology, and partial inflation indexation.

1. Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a major toolkit for empirical macroeconomic research and an important policy tool used in central banks. These models are firmly rooted in economic theory and can be derived mathematically by solving dynamic stochastic optimization problems with well-defined objective functions of the various agents (i.e. individuals, firms, financial intermediaries, fiscal and monetary authorities) as well as resource constraints. Many methods for solving and estimating DSGE models have been developed and used in order to obtain a detailed analysis and thorough estimation of dynamic macroeconomic relationships, see e.g. Fernández-Villaverde et al. (2016) for an overview. Recently, the question of identification of the parameters in DSGE models has proven to be of major importance, especially since identifiability precedes (consistent) estimation and inference of an unknown parameter vector from data. Parameter identification is a model property and can be analyzed by readily available diagnostic tools before actually taking a model to data. Canova and Sala (2009, p. 448) argue, however, that "DSGE models have never being built with an eye to the identification of their parameters". This paper can be interpreted as following up on their suggestion. We advocate to assess parameter identification from a model building perspective, as this provides a better understanding of the economic forces behind an identification result and, ultimately, behind the

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dynamics of the model before estimating it. To this end, we offer (1) a formal approach by using well-established diagnostics and indicators and (2) a set of applied tips on how to solve theoretical identification failures and improve the strength of DSGE model parameter identification by fine-tuning the functional specifications, model features and selection of observed variables and structural shocks.

Before we state the parameter identification problem formally, we briefly introduce our two example models to illustrate the problems at hand in a nontechnical way. The first example is taken from Kim (2003), who augments the canonical RBC model with both intertemporal and multisectoral investment adjustment costs. He then analytically shows that the two adjustment cost parameters, θ and κ, enter the linearized solution only through a composite parameter, $\frac{\theta+\kappa}{1+\theta}$, implying that they cannot be identified separately. We show how to solve this theoretical lack of identification by looking at (1) the selection of observed variables, (2) the functional specification of intertemporal adjustment costs, (3) additional model features like a cost on capital utilization and (4) additional structural shocks, namely an investment-specific technological innovation. Our second example is An and Schorfheide (2007), who consider a standard log-linearized New Keynesian model with government spending. Without going into the technical details yet, we already like to point out some obvious and non-obvious identification issues for illustrative purposes, as these are commonly shared by many other New Keynesian models. First, some steady state parameters, like steady state government spending or average technology, drop out from the linearized solution, so there is no way to make inference about them. Adding additional (e.g. measurement) equations may solve some of these issues but not all. Second, a common problem among linearized New Keynesian models is that there is an infinite number of combinations of the elasticity of demand, $1/\nu$, and price stickiness parameter, φ, which yield the exact same value for the slope κ of the New Keynesian Phillips curve. Hence, most empirical studies estimate κ instead. Third, the Taylor rule parameters are jointly not identifiable. For example, Qu and Tkachenko (2012, Table 1) show that the parameter combination $(\psi_\pi, \psi_y, \rho_R, \sigma_R^2) = (1.572, 0.001, 0.742, 0.391)$ yields the exact same model dynamics, moments and impulse responses as the parameter combination $(\psi_\pi, \psi_y, \rho_R, \sigma_R^2) = (0.992, 1.007, 0.796, 0.451)$, where ψ_π is the sensitivity to deviations of inflation from its target, ψ_y the sensitivity of the output-gap, ρ_R the persistence of the Taylor rule and σ_R the standard deviation of the monetary policy shock. Economically, this is a severe problem – especially for policy makers – as the first parametrization corresponds to a (hawkish) rule that responds only to inflation deviations, whereas the second parametrization mimics a (dovish) rule where the monetary authority balances its response due to deviations from both the inflation target and potential output. We show means to solve this theoretical lack of identification by looking at (1) the set of observed variables, (2) the functional specification of the output-gap, (3) additional model features like partial inflation indexation and (4) additional structural shocks, namely a preference shock on the discount factor.

Of course, uncovering such issues is not an easy task as analytical results are rarely available and feasible. But, there are several diagnostic tools which can help a researcher assess parameter identification. Nevertheless, even if parameters are theoretically identified, weak identification is a serious concern for applied macroeconomists. Identifiability, in this sense, is an empirical property dependent on the sample size. Therefore, this paper is also concerned with the sensitivity of fine-tuning model features on the strength of identification from a Bayesian point of view, as this has become the leading estimation paradigm in the literature. We aim to provide a practitioner's point of view on the complexities of assessing parameter identification in linearized DSGE models and make the available toolkit more accessible to a broader audience. Our research feeds into the ongoing development of the *identification toolbox* of Dynare (Adjemian et al., 2011), a widely used software platform to analyze, solve and estimate a wide class of economic models, such that our findings can be easily replicated and adapted to other models and needs. The replication files are available in a GitHub repository from the corresponding author, whereas our methodological contributions are already merged into Dynare's 4.6-unstable branch.

In section 2, we state the identification problem formally and the economic and econometric implications for DSGE models in more detail. We summarize our implementation of the used tools and provide guidance to uncover identifiability issues from an applied perspective in section 3. The fine-tuning of identification properties of the investment adjustment costs model is given in section 4, whereas section 5 provides the corresponding analysis for the monetary model. In section 6, we discuss the general implications of our results from a model building perspective and their relation to the current literature. Lastly, section 7 concludes.

2. The identification problem in DSGE models

Let $\theta \in \boldsymbol{\Theta}$ denote the (unknown) vector of model parameters, where $\boldsymbol{\Theta}$ is the admissible parameter space that yields a unique and stable solution, and $\boldsymbol{Y_T}$ the matrix of observable variables with sample size T. Further, $p(\theta; \boldsymbol{Y_T})$ denotes an objective function generated by a DSGE model, e.g. a probability distribution, likelihood, posterior or moment's distance. Following Rothenberg (1971),[1] $\boldsymbol{\theta}$ is said to be *locally identifiable* from p at a point $\boldsymbol{\theta_0} \in \boldsymbol{\Theta}$, if there exists an open neighborhood of $\boldsymbol{\theta_0}$ in which $p(\boldsymbol{\theta_0}; \boldsymbol{Y_T}) = p(\boldsymbol{\theta_1}; \boldsymbol{Y_T})$ implies $\boldsymbol{\theta_0} = \boldsymbol{\theta_1}$ for all $\boldsymbol{Y_T}$. The local point $\boldsymbol{\theta_0}$ usually corresponds to calibrated values, the maximum likelihood or minimal distance estimate, or the prior or posterior mean. In other words, identification problems arise if distinct parameter values do not lead to distinct objective functions of data, i.e. $p(\theta; \boldsymbol{Y_T})$ needs to be uniquely determined (in the injective sense) by $\boldsymbol{\theta_0}$. Even with an infinite sample, $T \to \infty$, it is not possible to pin down some (sets of) parameters, no matter what estimation procedure one uses. We refer to this as the *theoretical lack of identification*. By contrast, we are also concerned with the *empirical strength of identification*, i.e. how much information can be extracted from a specific $\boldsymbol{Y_T}$ to estimate model parameters. That is, even though all parameters enter the objective function separately and it has a unique extremum, its curvature may be small in certain regions of the parameter space, especially in small samples. We refer to this as *weak identification*. The literature is also concerned with *global identification*; however, it is numerically much more difficult to verify than local identification and therefore beyond the applied scope of this paper.[2]

From an economic point of view, lack of identification leads to wrong conclusions from calibration, estimation and inference (Canova and Sala, 2009), whereas the source of identification influences empirical findings (Ríos-Rull et al., 2012). From an econometric point of view, parameter identification belongs to the usual regularity conditions of commonly used estimators, e.g. the asymptotic theory of maximum likelihood requires local identification (Wald, 1949), whereas for the large sample properties of the generalized method of moments it is a necessary (but not sufficient) condition (Hansen, 1982). Accordingly, in a full-information setting this often evokes a badly shaped likelihood function with relatively flat regions, which modern Bayesian estimation can conveniently circumvent by using tight priors. The common notion, however, that "unidentifiability causes no real difficulties in the Bayesian approach" (Lindley, 1971, p. 46) is misleading and

[1] We refer to Aldrich (2002) for a historical overview of definitions of identifiability, especially in the Bayesian context.

[2] Qu and Tkachenko (2017) provide an algorithm that focuses on minimizing the Kullback-Leibler discrepancy from a frequency domain perspective, whereas Kocicecki and Kolasa (2018) exploit the link between observationally equivalent state space representations and the inherent constraints imposed by the model solution. Both approaches are computationally challenging and require a lot of fine-tuning. A model-independent implementation into Dynare is left for future research.

"it is necessary to clear up the ground from misunderstandings that may be detrimental for the methodology as a whole" (Canova, 2007, p. 191). In a nutshell, if parameters are not identifiable, the prior becomes extremely influential and needs to be informative for a proper posterior distribution. Moreover, in the case of prior dependence the comparison of prior and posterior for non-identified parameters can be misleading (Koop et al., 2013; Poirier, 1998) and may overstate the informativeness of the data about the parameters. Likewise, calibrating unidentified parameters can lead to wrong conclusions, since other parameters might depend on the calibrated ones, see Canova and Sala (2009) for an example.

Weak identification is likely to be a more serious concern for applied researchers. Accordingly, experience shows that it is quite difficult – both for Frequentists as well as Bayesians – to maximize the likelihood/posterior or minimize some (moment) distance function, because these functions are typically not well behaved and one has to deal with multiple local extrema, weak curvature in some directions of the parameter space and ridges. The evaluation of first-order and second-order derivatives is intractable and gradient based optimization methods perform quite poorly (Andreasen, 2010). The resulting estimators either hardly differ from initial values or may often lie on the boundary of the theoretically admissible parameter space which makes conventional Gaussian asymptotics a poor approximation to the true sampling distribution. In many cases the source of these peculiar outcomes is due to identifiability issues or an unfortunate choice of observables (Guerron-Quintana, 2010). Therefore, it is important to understand identification as a model property and check its sensitivity before taking a model to the data.

3. Implementation of identification checks

We are concerned with linearized DSGE models, i.e. we use first-order perturbation techniques to approximate the solution of a DSGE model as explained in Villemot (2011). We then carefully check the *rank criteria* of local identification of Iskrev (2010), Komunjer and Ng (2011) and Qu and Tkachenko (2012) for all considered model variants and sets of observables with Dynare. These three methods are the most basic and the closest to ideas from the early work on identification in systems of linear equations, since they are based on the uniqueness of a solution in the fashion of Rothenberg (1971). Identifiability, in this sense, is a theoretical property which can be analyzed before seeing any data. Regarding the strength of identification we follow Koop et al. (2013) who derive a *Bayesian learning rate indicator* to assess whether a parameter is strongly or weakly identifiable (and estimable). We now outline our implementation of the diagnostics and tools and elaborate on our method to select observable variables in more detail.

3.1. Rank checks

Iskrev (2010)'s approach to detect non-identified parameters is based on observational equivalent moments, i.e. to check whether the mapping from the parameter vector θ to the vector of theoretical first two moments (mean and autocovariances) is injective. In practice, a researcher needs to select the number of lags in the autocovariances (or autocorrelations). According to Ratto and Iskrev (2011), it suffices to check the rank condition for a small number of lags q, since the Jacobian is likely to have full rank for q much smaller than T. In most practical cases, q between 10 and 30 will be sufficient. A good candidate to try first is the smallest q for which the order condition is satisfied, and then increase the number of moments if the rank condition fails. To ease the computation of the rank, we advise to normalize the Jacobian by re-scaling each row by its largest element in absolute value.

Qu and Tkachenko (2012)'s approach focuses on observational equivalent spectral properties, i.e. on the sensitivity of the theoretical mean and spectrum of observables to changes in parameters. More precisely, the idea is to check whether the Hessian of the log-likelihood, when expressed as the outer product of the Jacobian matrix of derivatives of the spectral density with respect to θ, is full rank. Their criteria is therefore based on injectivity of the mapping from θ to the mean and to the spectral density. In practice, a researcher needs to select the number N of grid points in $[-\pi;\pi]$ to approximate the integral of the spectral density. Note that, if θ_0 is already identified from a subset of frequencies (small N), it is also identified if considering the full spectrum ($N \to \infty$) (the converse is not true). Therefore, we recommend starting with $N = 5000$ and increase N if the results are unsatisfactory. Moreover, in the code, we exploit symmetry and focus only on $[0;\pi]$ to speed up the computations. As the resulting Jacobian is a Gram-type matrix there is no order condition. Furthermore, we advise to normalize this matrix by transforming it into a correlation-type matrix with ones on the diagonal to ease the computation of the rank.

Komunjer and Ng (2011) study the implications of observational equivalence in minimal systems and derive a finite system of nonlinear equations that admit a unique solution if and only if the parameters are identified. They focus on the mapping from the model parameters to the state space representation, however, taking into account the possibility that the reduced-form parameters of the policy function may not be identifiable. Different to the moment and spectrum condition, this method does not require to compute the moments or spectral density explicitly. However, we need to find the smallest possible dimension of the state vector that is able to capture all dynamics and has the familiar state-space representation. Conceptually, as DSGE models are based upon microfoundations, this is not hard to determine for small and medium-sized DSGE models, e.g. in the code we first remove columns in the transition matrix that consist only of zeros and then run a brute-force search to find the minimal state vector. However, when dealing with auxiliary equations and variables this requires some more fine-tuning and user input, see e.g. Komunjer and Ng (2011, appendix) for illustrative examples. To ease the computation of the rank, we advise to normalize the Jacobian by re-scaling each row by its largest element in absolute value.

For all three diagnostics we need to compute a Jacobian with respect to model parameters and check whether it has full rank. Consequently, in the case of rank deficiency it is possible to pinpoint the (sets of) parameters that are locally indistinguishable from one another. In our experience, we find that the methods sometimes differ due to numerical settings, numerical errors or the method used to find problematic parameter sets. For instance, a researcher should try different tolerance levels to judge the robustness of the rank results. In this line of thought, Iskrev (2010) follows an analytical closed-form approach to compute the Jacobian of moments using Kronecker products, which is extended in Ratto and Iskrev (2011) by making use of computationally more efficient generalized Sylvester equations. Both Komunjer and Ng (2011) and Qu and Tkachenko (2012), however, rely on numerical methods to compute the derivatives of the minimal system and spectrum. Numerical differentiation is known to be very sensitive to the thresholds and tolerance levels used, see Mutschler (2016) for examples. Therefore, we extend Dynare's *identification toolbox* such that all three criteria (moments, minimal system and spectrum) are computed analytically and displayed by default. Under the hood, we extend ideas from Iskrev (2010) and Ratto and Iskrev (2011) in order to establish closed-form expressions for Komunjer and Ng (2011)'s and Qu and Tkachenko (2012)'s Jacobians using either Kronecker products or generalized Sylvester equations. Of course, a user may also choose to compute all Jacobians numerically and fine-tune the step size. In any case, it is important to use the same derivation method to improve comparability and robustness of an identification result. To pinpoint the problematic parameters that yield rank failure, the default in Dynare is to look into the null-space of the Jacobians and evaluate multi-correlation coefficients of the columns. Another approach, which is used in this paper, follows Qu and Tkachenko (2012, Corollary 4). That is, we check the rank criteria for all possible combinations of parameters in a recursive fashion and mark the ones that do not pass the rank check. In our experience, this brute-force approach yields more reliable results and is computationally just slightly slower, because, if we find a subset of parameters that are not identified, we can exclude that subset from higher-order subsets.

Our methodological contributions and improvements are already merged into the 4.6-unstable branch, such that an applied user can simply call the *identification* command on his/her model and adapt the options discussed in this paragraph.

3.2. Bayesian learning rate indicator

Koop et al. (2013) propose an indicator for weak identification based on the idea that the strength of identification becomes better as more data becomes available. In other words, the more data is used, the more precisely one can estimate a parameter, which implies shrinking posterior variances. This insight can be used to derive an indicator, that looks at the *average posterior precision* of the parameters, i.e. the inverse of the posterior variance divided by the sample size T. They show that the posterior precision should increase at a rate of T for strongly identified parameters, whereas for weakly identified parameters it increases at a slower rate. Therefore, the *average posterior precision* of a strongly identified parameter should tend to a constant, whereas for a weakly identified parameter it is heading towards zero.

We generate one artificial dataset of 50,000 observations and then use Dynare to estimate the parameters with Bayesian MCMC methods using the first $T = 100, 300, 900, 2700$ and 8100 of the simulated observations. Then, on the one hand, we follow the approach in Chadha and Shibayama (2018) and compute the average posterior precision by taking the inverse of the product of the posterior variance times T and examine if it converges to a constant, suggesting the posterior precision is updated at the same rate as T. On the other hand, we also compute convergence ratios as in Kamber et al. (2016); that is, we compare the ratio of two subsequent estimated posterior precision values, e.g. at $T = 100$ and $T = 300$, and check whether this ratio is close to the rate at which T increases, i.e. close to $300/100 = 3$.

Regarding the implementation, we heavily exploit Dynare's macro language and preprocessing capabilities to loop over sample sizes and fine-tune the estimation commands for the different model variants. Following common practice, we use a Random-Walk Metropolis-Hastings sampling algorithm based on four Markov chains with each 1,000,000 draws, half are being discarded as burn-in draws in each chain. The mode and Hessian evaluated at the mode (computed by Dynare's mode_compute = 4, i.e. Chris Sims's CSMINWEL) are used to determine the initial Gaussian proposal density with scale parameter set such that the acceptance ratios lie in between 20% and 35%. In some cases, we use an advanced mode finding procedure, where we sequentially loop over different optimization algorithms taking the previous found mode as initial value for the next optimizer. In particular, we loop, in this order, over Dynare's mode_compute values equal to 9 (CMA-ES), 8 (Nelson-Mead Simplex), 4 (csminwel), 7 (fminsearch) and 1 (fmincon). We then rely on Dynare's (very time-consuming) mode_compute = 6 optimizer, i.e. a "Monte Carlo Optimizer" to get a well-behaved Hessian in the relevant parameter space. The intuition is that the Metropolis-Hastings algorithm does not need to start from the posterior mode to converge to the posterior distribution. It is only required to start from a point with a high posterior density value and to use an estimate of the covariance matrix for the jumping distribution (actually any positive definite matrix suffices).[3] All estimation results and convergence diagnostics are available in the replication files.

3.3. Selection of observables

Ideally, economic intuition dictates the selection of observables that reveal the most useful information about the parameters of interest. For example, in models with monetary neutrality, we know that nominal variables have no real effects, so this needs to be taken into account when selecting observable variables. Similarly, Martínez-García et al. (2012) find that observing the terms of trade improves the strength of identification in an open economy model. Likewise, Andreasen and Dang (2019) show that the price demand elasticity can be estimated reliably in a standard log-linearized version of the New Keynesian model when including firm profit as an observable in the estimation. However, Canova et al. (2014) warn that there are important trade-offs when deciding to use hours or labor productivity together with output among the observables in a variant of the Smets and Wouters (2007) model. They caution that different combinations of variables may produce different responses to shocks. A point echoed by Martínez-García et al. (2012) and Martínez-García and Wynne (2014) who additionally raise the issue of data availability limitations in practice.

So, unfortunately, there is no general guideline on selecting observables, as one needs to experiment with a number of possible and feasible combinations of variables. Therefore, in our exercise, we are agnostic and use a brute-force approach, i.e. we check all three diagnostics for all possible combinations of observable variables. This basically mimics Canova et al. (2014), who select observables in a way that optimizes parameter identification according to Komunjer and Ng (2011)'s rank criteria. As outlined above, this diagnostic is not always available; hence, we also optimize along the lines of Iskrev (2010)'s moment and Qu and Tkachenko (2012)'s spectrum rank criteria for a robust comparison. After having established which sets of observables are theoretically favored in terms of local identification, we then use economic hindsight to choose feasible sets and run the identification strength indicator on these.

4. Investment adjustment costs model

4.1. Model description

The Kim (2003) model is a variant of the canonical Real Business Cycle model with log utility extended by two kinds of investment adjustment costs. First, *multisectoral adjustment costs*, governed by a parameter θ, enter the budget constraint:

$$\underbrace{\left[(1-SAV)\left(\frac{C_t}{1-SAV}\right)^{1+\theta}+SAV\left(\frac{I_t}{SAV}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}}}_{:=Y_t^d} = R_t^K U_t^K K_{t-1} - \Psi_t^K K_{t-1} \quad (1)$$

where C_t is consumption, I_t is investment and $SAV = \frac{I}{Y^d}$ denotes the steady state savings rate. Similar to Huffman and Wynne (1999) we focus on $\theta > 0$, i.e. a reverse CES technology, in order for the production possibilities set to be convex. Note that for $\theta = 0$ the transformation reduces to the standard linear case, i.e. demand Y_t^d is equal to consumption and investment. Different to Kim (2003), we introduce a cost, Ψ_t^K, of capital utilization per unit of physical capital. U_t^K denotes the capital utilization rate and we use the following functional form: $\Psi_t^K = (1-\psi_K)(U_t^K - U^K) + \frac{\psi_K}{2}(U_t^K - U^K)^2$, such that the usual steady state normalization, $\Psi^{K''}/\Psi^{K'} = \psi_K/(1-\psi_K)$, applies. Physical (end-of-period) capital, K_t, is transformed into effective (end-of-period) capital, K_t^s, according to $K_{t-1}^s = U_t^K K_{t-1}$. Effective capital is then rented to the representative firm at the gross rental rate R_t^K. The firm pro-

[3] The replication files of An and Schorfheide (2007) reveal that they face the same problem in their estimation and overcome this by using different step sizes for the numerical evaluation of second derivatives of the log-likelihood function. Other common "tricks" to overcome a singular Hessian are to decompose the Hessian, set the eigenvalues smaller than or equal zero to some small number, and then recompose it. We thank Johannes Pfeifer for pointing this out.

duces a homogeneous good using a Cobb-Douglas production function, $Y_t = A_t(K^s_{t-1})^\alpha$, where A_t denotes total factor productivity.

Second, *intertemporal adjustment costs*, governed by a parameter κ, are introduced into the capital accumulation equation, which involve a nonlinear substitution between the capital stock and investment. We consider two different specifications for this friction:

$$K_t = \left[(1-\delta)K_{t-1}^{1-\kappa} + \delta\left(\frac{\upsilon_t I_t}{\delta}\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}} \tag{2a}$$

$$K_t = (1-\delta)K_{t-1} + \upsilon_t I_t\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) \tag{2b}$$

where δ denotes the depreciation rate and we set $S_t := S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, such that the usual steady state normalization, $S_t(1) = 0$, $S_t'(1) = 0$ and $S_t''(1) > 0$, applies. Equation (2a), which we call the LEVEL specification, is also used by Kim (2003). It is based on Lucas and Prescott (1971) and involves costs in terms of the first derivative of capital or, in other words, on the current level of investment. Equation (2b), which we call the GROWTH specification, is based on Christiano et al. (2005) and involves costs in terms of investment changes between periods. Note that for $\kappa = 0$ we get the usual linear capital accumulation specification, i.e. $K_t = (1-\delta)K_{t-1} + \upsilon_t I_t$, in both cases. Different to Kim (2003), we introduce investment-specific technological change, υ_t, in the fashion of Greenwood et al. (2000) and Justiniano et al. (2010). Both the log of A_t and the log of υ_t evolve according to AR(1) processes with persistence ρ_j and additive shocks, $\epsilon_{j,t}$, which are assumed to be normally distributed with zero mean and standard deviation σ_j ($j = A, \upsilon$). The model equations are summarized in Table 1, where $\beta = 1/(1 + R^A/400)$ and Λ_t and $\Lambda_t Q_t$ denote the Lagrange multipliers corresponding to equations (1) and (2) respectively. The upper parts of the equations correspond to the LEVEL specification of intertemporal investment adjustment costs, whereas the lower parts are associated with the GROWTH specification. The steady state is given by normalizations, $A = Q = \upsilon = 1$, and equations $R^K = (1-\psi_K) = \left(\frac{1}{\beta} + \delta - 1\right)\frac{Q}{U^K}$, $K = \left(\frac{\alpha A}{R^K}\right)^{\frac{1}{1-\alpha}}$, $I = \frac{\delta K}{\upsilon}$, $Y = AK^\alpha$, $C = (1 - SAV)Y$ and $\Lambda = C^{-1}$. The calibration of α, β and δ is based on a steady state investment to output ratio, I/Y, of 0.25, a steady state capital productivity, K/Y, of 10 and an annualized steady state interest rate, R^A, of 2. ψ_K is implicitly defined via the first order necessary conditions with respect to K_t and U^K_t. θ and κ are based on values taken from Ratto and Iskrev (2011), whereas the parameters of the stochastic processes are chosen symmetrically with mild persistence and amplitude of shocks. The calibration and prior specification of parameters is summarized in Table 2.

4.2. Model variants

To check the sensitivity of local identifiability to changes in observables, model assumptions, functional specifications and shocks, we distinguish three different model scenarios and consider all possible one- and two-set combinations of model variables as observables. Our focus lies on observable variables that are commonly used in the literature; namely, output, consumption, investment and the return of capital.[4] Our first scenario, called BASELINE, corresponds to the original model specification of Kim (2003). Accordingly, we switch off both capital utilization and investment-specific technological change. In our second scenario, called CAPITAL UTILIZATION, we analyze the effect on local identification of adding capital utilization costs to the BASELINE scenario. Likewise, in our third scenario, called INVESTMENT SHOCK, we add investment-specific technological change to the BASELINE case. Note that in the first two scenarios there is only one structural shock in the model, whereas in the last scenario there are two. Lastly, each scenario is run with either the LEVEL or GROWTH specification of intertemporal investment adjustment costs. The following sensitivity analysis of identification as a model property is based on the calibrated local point $\boldsymbol{\theta_0}$ given in the second column of Table 2. The replication files also contain the local identifiability analysis for the prior mean as well as 100 random draws from the prior domain. As the results are almost identical for all model variants, we focus on the calibrated values in our exposition.

4.3. Theoretical identification

Table 3 summarizes whether the required rank conditions are fulfilled for the different scenarios and combinations of observable variables at $\boldsymbol{\theta_0}$. As expected and analytically shown by Kim (2003), θ and κ cannot be identified jointly in the BASELINE scenario with the LEVEL specification. The GROWTH specification, however, allows one to identify these parameters for many choices of feasible observable variables. In particular, among single observable variables, consumption C_t (and not output Y_t) yields a locally fully identified model. Intuitively, the GROWTH specification adds another state variable into the model in terms of lagged investment. The coefficients of lagged investment in the decision rules depend on the intertemporal adjustment costs parameter κ in a manner that is distinct from the multisectoral adjustment costs parameter θ. Hence, we can distinguish the dynamics of multisectoral *level* adjustment costs from intertemporal *growth* adjustment costs. Our other two scenarios, CAPITAL UTILIZATION and INVESTMENT SHOCK, individually introduce sufficient internal dynamics into the optimal allocation of investment and capital. These features tend to smooth the adjustment of the rental rate of capital, and therefore, enhance identifiability of adjustment cost parameters κ and θ. Almost all pairs of variables yield full rank in these scenarios, independent of whether we consider the LEVEL or the GROWTH specification. Note that single observable variables fail to identify all model parameters as κ and θ are co-linear with either the capital utilization or the investment shock process parameters.

Moreover, in the replication files we also consider the effect of a different utility function, internal or external habit, labor choice and monetary policy rules on parameter identification of θ and κ. We briefly summarize our findings. A CRRA utility function or the inclusion of internal/external habit formation does not change the above results.[5] The inclusion of labor (as already shown by Kim (2003)) facilitates parameter identification of θ and κ in both cases, but adds other parameters that can only be identified by observing either hours or wages. Extending the BASELINE model with respect to bond holdings requires the inclusion of a Taylor rule. This also provides means for identifying the investment adjustment costs parameters in both the LEVEL and GROWTH specification, however, for several combinations of observables the parameters of the monetary rule are not identified, a topic we study in more detail in section 5.

4.4. Weak identification

Tables 4 and 5 provide insight into the strength of identification

[4] For the sake of completeness, we also analyze (usually unobserved) variables like technology, capital or the auxiliary Lagrange multipliers. We like to point out that by observing marginal utility Λ_t or Tobin's Q_t combined with any other variable, one is able to locally identify all model parameters independent of the specification of intertemporal investment adjustment costs or model scenario. Observing capital or technology, on the other hand, does not solve the lack of identification in the considered scenarios. The exact results can be found in the replication files.

[5] In some cases we find that the identification criteria yield different results. We experimented with the settings and found that the differences are driven by numerical thresholds, tolerance levels and the method used to normalize the Jacobians for rank computations.

Table 1
Model equations of investment adjustment costs model.

$$\left(\frac{C_t}{(1-SAV)\cdot Y_t^d}\right)^{\theta}\Lambda_t = C_t^{-1}$$
$$\left(\frac{I_t}{SAV\cdot Y_t^d}\right)^{\theta}\Lambda_t = \begin{cases}\Lambda_t Q_t\left(\frac{\delta K_t}{\upsilon_t I_t}\right)^{\kappa}\\ \Lambda_t Q_t\left(1-S_t-\left(\frac{I_t}{I_{t-1}}\right)S_t'\right)\upsilon_t+\beta E_t\left[\Lambda_{t+1}Q_{t+1}\upsilon_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S_{t+1}'\right]\end{cases}$$
$$\Lambda_t Q_t = \begin{cases}\beta E_t\left[\Lambda_{t+1}\left(R_{t+1}^K U_{t+1}^K-\Psi_{t+1}^K+(1-\delta)Q_{t+1}\left(\frac{K_{t+1}}{K_t}\right)^{\kappa}\right)\right]\\ \beta E_t\left[\Lambda_{t+1}\left(R_{t+1}^K U_{t+1}^K-\Psi_{t+1}^K+(1-\delta)Q_{t+1}\right)\right]\end{cases}$$
$$R_t^K = \Psi_t^{K'}$$
$$R_t^K = \frac{\alpha Y_t}{K_{t-1}^S}$$
$$Y_t = Y_t^d + \Psi_t^K K_{t-1}$$
$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$$
$$\log \upsilon_t = \rho_\upsilon \log \upsilon_{t-1} + \varepsilon_t^\upsilon$$

Table 2
Parameters, priors and bounds for investment adjustment costs model.

Parameters		Bounds		Prior Specification		
Symbol	θ_0	Lower	Upper	Density	Mean	Std. deviation
θ	1.5	1e-8	10	Gamma	1.5	0.75
κ	2	1e-8	10	Gamma	2	1.5
α	0.3	1e-8	0.9999	Normal	0.3	0.05
δ	0.025	1e-8	0.9999	Uniform	0	1
R^A	2	1e-8	10	Gamma	2	0.25
ρ_A	0.5	1e-8	0.9999	Beta	0.5	0.1
σ_A	0.6	1e-8	10	Inv Gamma	0.6	2
ψ_K	0.97	1e-8	0.9999	Uniform	0	1
ρ_υ	0.5	1e-8	0.9999	Beta	0.5	0.1
σ_υ	0.6	1e-8	10	Inv Gamma	0.6	2

according to the Bayesian learning rate indicator of Koop et al. (2013) for the BASELINE scenario with observable C_t and the INVESTMENT SHOCK scenario with observable Y_t and C_t. We choose these scenarios due to the fact that our focus is on applied researchers who use Dynare for Bayesian estimation. Accordingly, we do not analyze the strength of identification in the CAPITAL UTILIZATION scenario as this requires techniques to estimate singular DSGE models, which cannot be done with Dynare out-of-the-box (yet). The simulation and estimation exercise reveals that the strength of identification of the investment adjustment costs parameters, θ and κ, is weak in both BASELINE LEVEL and BASELINE GROWTH scenarios as well as the INVESTMENT SHOCK LEVEL case, since the rates at which the posterior precisions are updated are slower than the sample size change. That is, the average posterior precision values in Table 4 tend towards zero instead of a constant value. This is also evident by looking at the convergence ratios in Table 5 as these stay close to 1 and do not tend towards the change in sample size of 3. The INVESTMENT SHOCK GROWTH specification, however, is the exception, as κ and θ are both strongly identifiable: The average precisions tend towards a constant and the convergence ratios fluctuate around 3. Regarding the other model parameters we find mixed results. In all cases under consideration the strength of identification of R^A (and hence β) is weak, which is a common finding in the literature (Morris, 2017). In the (unidentified) BASELINE LEVEL scenario we see that only ρ_A is strongly identifiable. This confirms that estimating non-identified models yields severe problems in the estimation of other, actually identified model parameters. Accordingly, in the (identified) BASELINE GROWTH scenario α, δ, ρ_A and σ_A are (more or less) strongly identified. Likewise, the GROWTH specification in the INVESTMENT SHOCK scenario performs better than the LEVEL one as the convergence ratios are closer to 3.

4.5. Summary

To sum up, in all our experiments we find that the GROWTH specification of intertemporal investment adjustment costs is superior to the LEVEL specification in terms of theoretical identification. Moreover, in the BASELINE GROWTH scenario, there is a single best choice as observable: consumption (and not output or investment) is able to locally identify all parameters. Lastly, investment-specific technological change improves the strength of model parameters. Therefore, we provide theoretical support (from an identification point-of-view) for using both Christiano et al. (2005)'s GROWTH specification of investment adjustment costs and Greenwood et al. (2000)'s investment-specific technological change in modern DSGE models.

5. Monetary model

5.1. Model description

The An and Schorfheide (2007) model is a prototypical New Keynesian DSGE model and consists of a representative household purchasing a basket of differentiated goods using a Dixit-Stiglitz type aggregator and supplying homogeneous labor services. The differentiated goods are supplied by monopolistically competitive firms using only labor services according to a linear production function. Each firm sets prices conforming to the Rotemberg pricing assumption such that changing prices entails a real cost in terms of goods. Labor productivity, A_t, is the driving force of the economy and evolves according to a unit root process, i.e. $\log(A_t/A_{t-1}) = \log(\gamma) + \log(z_t)$, where γ denotes the steady state growth rate of the economy. Hence, $y_t = Y_t/A_t$ stands for detrended output and $c_t = C_t/A_t$ for detrended consumption. The

Table 3
Rank checks for investment adjustment costs model.

	Baseline						Capital Utilization						Investment Shock					
	Level			Growth			Level			Growth			Level			Growth		
	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC	MOM	MIN	SPEC
Y	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]
C	[$\kappa\theta$]	ERR	[$\kappa\theta$]	✓✓	✓✓	✓✓	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	✓✓	ERR	[$\kappa\theta$]
I	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]
R^K	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[κ]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	✓✓	ERR	[$\kappa\theta$]
K	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]
Λ	[$\kappa\theta$]	ERR	[$\kappa\theta$]	✓✓	✓✓	✓✓	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	✓✓	ERR	[$\kappa\theta$]
Q	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]
A	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	ERR	[$\kappa\theta$]
U^K	–	–	–	–	–	–	[$\kappa\theta$]	ERR	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	–	–	–	–	–	–
υ	–	–	–	–	–	–	–	–	–	–	–	–	[$\kappa\theta$]	err	[$\kappa\theta$]	[$\kappa\theta$]	err	[$\kappa\theta$]
Y,C	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓
Y,I	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓
Y,R^K	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓
C,I	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓
C,R^K	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓
I,R^K	[$\kappa\theta$]	[$\kappa\theta$]	[$\kappa\theta$]	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓

Notes: MOM corresponds to Iskrev (2010)'s, MIN to Komunjer and Ng (2011)'s and SPEC to Qu and Tkachenko (2012)'s rank criteria. A ✓✓ indicates that all model parameters are theoretically identifiable at $\boldsymbol{\theta_0}$ given in Table 2. [$\kappa\theta$] indicates that both κ and θ cannot be identified jointly or are co-linear with respect to other parameters, whereas a single [κ] or [θ] implies non-identification of that parameter. ERR indicates that the criteria cannot be computed (mostly the order condition is not met), whereas a - indicates that this set of variables is not available in the specific scenario.

monetary authority follows a Taylor rule for the nominal interest rate R_t and real government spending G_t is assumed to evolve stochastically as a ratio of output $g_t := (1 - G_t/Y_t)^{-1}$. Uncertainty is introduced via random fluctuations in productivity growth, government spending and a monetary policy shock.

We extend the model in three common directions. First, we add a preference shock, ζ_t, to the utility function that shifts the discount factor in the intertemporal optimization problem of the household without changing the intratemporal labor supply decision. Therefore, the detrended Lagrange multiplier corresponding to marginal consumption utility is given by $\lambda_t = \zeta_t c_t^{-\tau}$. Second, the Rotemberg price adjustment function of the j-th intermediate firm, $ac_t(j) = \frac{\varphi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \Gamma_{t-1}\right)^2 y_t(j)$, follows either a full or a partial indexation scheme:

$$\Gamma_{t-1} = \pi^* \quad \text{(full indexation)} \tag{3a}$$

$$\Gamma_{t-1} = \pi_{t-1}^{\iota^P} \pi^{*1-\iota^P} \quad \text{(partial indexation)} \tag{3b}$$

where π^* denotes target inflation. The first scheme corresponds to the original specification of An and Schorfheide (2007), whereas the second one is in the fashion of Smets and Wouters (2007) or Born and Pfeifer (2019). Third, we consider four different monetary policy rules that differ in the definition of the output-gap: (i) deviation from the output value under flexible prices but with the monopoly power distortion intact, (ii) deviation from the steady state value of output, (iii) deviation from the growth trend and (iv) the Smets and Wouters (2007) rule which combines (i) with differences in growth rates of output and the flex-price output:

$$R_t^*/R = \left(\pi_t/\pi^*\right)^{\psi_\pi} \left(y_t/y_t^*\right)^{\psi_y} \quad \text{(flex – price)}$$

$$R_t^*/R = \left(\pi_t/\pi^*\right)^{\psi_\pi} (y_t/y)^{\psi_y} \quad \text{(steady state)} \tag{4b}$$

$$R_t^*/R = \left(\pi_t/\pi^*\right)^{\psi_\pi} (z_t \cdot y_t/y_{t-1})^{\psi_y} \quad \text{(growth)} \tag{4c}$$

$$R_t^*/R = \left(\pi_t/\pi^*\right)^{\psi_\pi} \left(y_t/y_t^*\right)^{\psi_y} \left(\frac{y_t/y_{t-1}}{y_t^*/y_{t-1}^*}\right)^{\psi_{\Delta y}} \quad \text{(SW)} \tag{4d}$$

Note that $\pi^* = 1 + \pi^A/400$ is the target inflation rate and $y_t^* = (1-\nu)^{\frac{1}{\tau}} g_t$ the output under flexible prices ($\varphi = 0$) but with the monopoly power distortion intact. All shocks, $\varepsilon_{j,t} (j = R, g, z, \zeta)$, are assumed to be normally distributed with zero mean and standard deviation σ_j. The model equations are summarized in Table 6, where $\gamma = 1 + \gamma^Q/100$, $\beta = (1 + r^A/400)^{-1}$ and we added measurement equations for the quarterly output growth rate YGR_t, the annualized inflation rate $INFL_t$ and the annualized interest rate INT_t.

The steady state is given by normalizations, $z = \zeta = 1$, $\pi = \pi^*$, $g = g^*$, and equations $R = \frac{\gamma z \pi}{\beta}$, $c = (1-\nu)^{\frac{1}{\tau}}$, $y = g \cdot c$, $YGR = \gamma^Q$, $INFL = \pi^A$, $INT = \pi^A + r^A + 4\gamma^Q$. The calibration, priors and bounds for the model parameters are summarized in Table 7 and are taken from An and Schorfheide (2007) and Smets and Wouters (2007). Note that a log-linearization and straightforward manipulation of equations yield the following New Keynesian IS and Phillips curves:

$$\hat{y}_t - \hat{g}_t = E_t[\hat{y}_{t+1}] - E_t[\hat{g}_{t+1}] - \frac{1}{\tau}\left(\hat{R}_t - E_t[\hat{\pi}_{t+1}] - E_t[\hat{z}_{t+1}] + E_t[\hat{\zeta}_{t+1}] - \hat{\zeta}_t\right) \tag{5}$$

$$\left(\hat{\pi} - \hat{\Gamma}_{t-1}\right) = \beta\left(E_t[\hat{\pi}_{t+1}] - \hat{\Gamma}_t\right) + \underbrace{\tau\frac{(1-\nu)}{\pi^{*2}\nu\varphi}}_{=:\kappa}(\hat{y}_t - \hat{g}_t) \tag{6}$$

where a hat variable denotes log deviations from steady state. In the case of full inflation indexation, $\hat{\Gamma}_{t-1} = \hat{\Gamma}_t = 0$, the New Keynesian

Table 4
Average posterior precisions for investment adjustment costs model.

T	α	R^A	δ	ρ_A	σ_A	θ	κ	ρ_υ	σ_υ
				BASELINE LEVEL, OBSERVABLE $[C_t]$					
100	4.95629	0.16204	63.98723	2.34967	2.21339	0.01792	0.00464	–	–
300	1.66085	0.05333	10.30643	1.65848	0.37558	0.00584	0.00235	–	–
900	0.58057	0.01758	4.48603	1.36247	0.25473	0.00196	0.00062	–	–
2700	0.21430	0.00584	2.30263	1.27458	0.15934	0.00067	0.00015	–	–
8100	0.09105	0.00199	1.22613	1.32859	0.07182	0.00024	0.00004	–	–
				BASELINE GROWTH, OBSERVABLE $[C_t]$					
100	5.12340	0.15921	70.38120	2.16254	2.45120	0.01729	0.00442	–	–
300	2.01095	0.05354	13.80977	1.39984	0.43133	0.00592	0.00151	–	–
900	0.74712	0.01764	6.19138	0.81634	0.25277	0.00198	0.00042	–	–
2700	0.32316	0.00599	3.50164	0.58203	0.19985	0.00071	0.00018	–	–
8100	0.20646	0.00197	3.64481	0.38089	0.10312	0.00021	0.00007	–	–
				INVESTMENT SHOCK LEVEL, OBSERVABLE $[Y_t, C_t]$					
100	79.65821	0.16049	248.06544	2.20163	0.95563	0.01990	0.05570	2.06671	0.73529
300	60.99857	0.05292	203.41839	1.52785	1.06409	0.00624	0.03569	1.57389	0.22976
900	36.45517	0.01786	199.70719	1.35222	1.16961	0.00220	0.01350	1.39302	0.12932
2700	17.90353	0.00598	99.03021	1.33281	0.79819	0.00088	0.00465	1.31428	0.06369
8100	7.19273	0.00206	61.85160	1.33369	0.83536	0.00035	0.00183	1.31305	0.02693
				INVESTMENT SHOCK GROWTH, OBSERVABLE $[Y_t, C_t]$					
100	70.43775	0.16351	309.84595	2.33406	0.90310	0.18742	0.18493	1.85570	1.22209
300	59.33852	0.05270	231.00262	1.57822	0.93599	0.06849	0.11995	1.39053	0.50447
900	36.56057	0.01785	227.14969	1.38118	1.07566	0.04748	0.09676	1.30805	0.43374
2700	17.79301	0.00617	116.20152	1.36056	0.73564	0.05269	0.09986	1.26627	0.38693
8100	7.68396	0.00242	84.64285	1.35743	0.82205	0.04133	0.08485	1.26177	0.34846

Table 5
Convergence ratios for posterior precisions for investment adjustment costs model.

dT	α	R^A	δ	ρ_A	σ_A	θ	κ	ρ_υ	σ_υ
				BASELINE LEVEL, OBSERVABLE $[C_t]$					
300/100	1.005	0.987	0.483	2.118	0.509	0.978	1.520	–	–
900/300	1.049	0.989	1.306	2.465	2.035	1.008	0.793	–	–
2700/900	1.107	0.997	1.540	2.806	1.877	1.032	0.742	–	–
8100/2700	1.275	1.021	1.597	3.127	1.352	1.051	0.836	–	–
				BASELINE GROWTH, OBSERVABLE $[C_t]$					
300/100	1.178	1.009	0.589	1.942	0.528	1.027	1.025	–	–
900/300	1.115	0.989	1.345	1.750	1.758	1.003	0.845	–	–
2700/900	1.298	1.018	1.697	2.139	2.372	1.073	1.273	–	–
8100/2700	1.917	0.985	3.123	1.963	1.548	0.911	1.098	–	–
				INVESTMENT SHOCK LEVEL, OBSERVABLE $[Y_t, C_t]$					
300/100	2.297	0.989	2.460	2.082	3.340	0.941	1.922	2.285	0.937
900/300	1.793	1.013	2.945	2.655	3.297	1.056	1.134	2.655	1.689
2700/900	1.473	1.005	1.488	2.957	2.047	1.209	1.033	2.830	1.477
8100/2700	1.205	1.034	1.874	3.002	3.140	1.186	1.180	2.997	1.269
				INVESTMENT SHOCK GROWTH, OBSERVABLE $[Y_t, C_t]$					
300/100	2.527	0.967	2.237	2.029	3.109	1.096	1.946	2.248	1.238
900/300	1.848	1.016	2.950	2.625	3.448	2.080	2.420	2.822	2.579
2700/900	1.460	1.037	1.535	2.955	2.052	3.329	3.096	2.904	2.676
8100/2700	1.296	1.175	2.185	2.993	3.352	2.353	2.549	2.989	2.702

Phillips curve is forward-looking, whereas partial inflation indexation adds a backward-looking component, as $\widehat{\Gamma}_{t-1} = \iota^p \widehat{\pi}_{t-1}$. Now, the issues we discuss in the introduction become obvious. That is, ν and φ are not independent parameters, there is an infinite number of combinations of the elasticity of demand, $1/\nu$, and the price stickiness parameter, φ, which yield the exact same value for the slope κ of the New Keynesian Phillips curve. Likewise, the steady state government spending target g^* does not enter the linearized solution. As our focus in this section is on the monetary policy parameters, we fix φ and g^* for now and discuss possible ways to identify ν, φ and g^* in the summary subsection.

5.2. *Model variants*

In our experiments, we distinguish three different model scenarios. Our first scenario, called BASELINE, corresponds to the original model specification of An and Schorfheide (2007). Accordingly, we consider full inflation indexation and switch off the discount factor shifter. In our second scenario, called PARTIAL INDEXATION, we analyze the effect of adding the partial inflation indexation scheme to the BASELINE scenario. In our third scenario, called PREFERENCE SHOCK, we add the discount factor shifter to the BASELINE model. We run all scenarios under the four

Table 6
Model equations of monetary model.

$$\lambda_t = \zeta_t c_t^{-\tau}$$
$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{R_t}{\gamma z_{t+1} \pi_{t+1}} \right]$$
$$1 = \frac{1}{\nu} \left(1 - (\lambda_t/\zeta_t)^{-1}\right) + \varphi(\pi_t - \Gamma_{t-1})\pi_t - \frac{\varphi}{2\nu}(\pi_t - \Gamma_{t-1})^2 - \varphi\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \Gamma_t)\pi_{t+1} \right]$$
$$y_t = c_t + \left(1 - \frac{1}{g_t}\right) y_t + \frac{\varphi}{2}(\pi_t - \Gamma_{t-1})^2 y_t$$
$$\log(g_t) = (1 - \rho_g)\log(g) + \rho_g \log(g_{t-1}) + \varepsilon_{g,t}$$
$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}$$
$$\log(\zeta_t) = (1 - \rho_\zeta)\log(\zeta) + \rho_\zeta \log(\zeta_{t-1}) + \varepsilon_{\zeta,t}$$
$$R_t = {R_t^*}^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\varepsilon_{R,t}\}$$
$$YGR_t = \gamma^Q + 100\left[\log\left(\frac{y_t}{y_{t-1}}\right) + \log\left(\frac{z_t}{z}\right)\right]$$
$$INFL_t = \pi^A + 400\log\left(\frac{\pi_t}{\pi}\right)$$
$$INT_t = \pi^A + r^A + 4\gamma^Q + 400\log\left(\frac{R_t}{R}\right)$$

Table 7
Parameters, priors and bounds for monetary model.

Parameters		Prior Specification			Bounds	
Symbol	θ_0	Density	Mean	Std. Deviation	Lower	Upper
r^A	1.00	Gamma	0.80	0.50	1e-5	10
π^A	3.20	Gamma	4.00	2.00	1e-5	20
γ^Q	0.55	Normal	0.40	0.20	−5	5
τ	2.00	Gamma	2.00	0.50	1e-5	10
ν	0.10	Beta	0.10	0.05	1e-5	0.99999
ψ_π	1.50	Gamma	1.50	0.25	1e-5	10
ψ_y	0.125	Gamma	0.50	0.25	1e-5	10
$\psi_{\Delta y}$	0.2	Gamma	0.20	0.15	1e-5	10
ρ_R	0.75	Beta	0.50	0.20	1e-5	0.99999
ρ_g	0.95	Beta	0.80	0.10	1e-5	0.99999
ρ_z	0.90	Beta	0.66	0.15	1e-5	0.99999
$100\sigma_R$	0.2	Inv Gamma	0.30	4.00	1e-8	5
$100\sigma_g$	0.6	Inv Gamma	0.40	4.00	1e-8	5
$100\sigma_z$	0.3	Inv Gamma	0.40	4.00	1e-8	5
ι^p	0.5	Beta	0.50	0.15	1e-8	1
ρ_ζ	0.75	Beta	0.50	0.20	1e-5	0.99999
$100\sigma_\zeta$	0.2	Inv Gamma	0.30	4.00	1e-8	5
φ	50	–	–	–	–	–
$1/g^*$	0.85	–	–	–	–	–

different monetary policy rules. We only report the results for observable variables YGR_t, $INFL_t$ and INT_t here, as, on the one hand, we find that γ_Q can only be identified from observing YGR_t, and, on the other hand, other combinations of model variables do not change our results significantly. We refer to the replication files for the full set of results, i.e. for all possible combinations of up to three variables. The following sensitivity analysis of identification as a model property is based on the calibrated local point θ_0 given in the second column of Table 7. The replication files also contain the local identifiability analysis for the prior mean as well as 100 random draws from the prior domain. As the results are almost identical for all model variants, we focus on the calibrated values in our exposition.

5.3. Theoretical identification

Table 8 summarizes whether the rank requirements are fulfilled for the different scenarios and monetary policy rules at θ_0. As shown by e.g. Komunjer and Ng (2011) or Qu and Tkachenko (2012), the monetary policy parameters (ψ_y, ψ_π, ρ_R, σ_R) cannot be identified in the BASELINE specification when using the FLEX-PRICE or the SW monetary rule, whereas in the STEADY STATE or GROWTH specifications these parameters are locally identifiable. Our analysis shows two more ways to solve the lack of identification, which are, moreover, independent of the functional form of the output-gap: adding a partial inflation indexation scheme and/or a preference shock.[6] Intuitively, the introduction of partial indexation results in a dynamic inflation specification in equation (6) that will also depend on past inflation, where the degree of indexation determines how backward looking the inflation process is. This, of course, has an effect on the transmission channel of monetary policy, as the price dispersion between individual prices of the monopolistic competitors will be much smaller compared to a constant price setting behavior. In contrast, the preference shock basically resembles a demand shock, as it shifts the effective discount factor that determines the intertemporal substitution decisions of households. According to the IS curve in equation (5) and the Phillips curve in equation (6), a positive impulse in the preference shifter leads to a positive impact on consumption and output, but also to some inflationary pressures and a partial crowding out of investment, see also Smets and Wouters (2003) for a similar result. Therefore, the overall effect on the output-gap and on the nominal interest rate add sufficient internal dynamics to the transmission channel of monetary policy to identify the Taylor rule parameters in all scenarios separately.

5.4. Weak identification

Tables 9 and 10 give insight into the strength of identification

[6] Kocicecki and Kolasa (2018) also show that spillovers from public spending to productivity can be an alternative way to ensure local identification in the BASELINE FLEX-PRICE scenario.

Table 8
Rank checks for monetary model.

Scenario	Monetary Policy Specification			
	FLEX-PRICE	STEADY STATE	GROWTH	SW
BASELINE	$[\psi_\pi, \psi_y, \rho_R, \sigma_R]$	✓✓	✓✓	$[\psi_\pi, \psi_y, \rho_R, \sigma_R]$
PARTIAL INDEXATION	✓✓	✓✓	✓✓	✓✓
PREFERENCE SHOCK	✓✓	✓✓	✓✓	✓✓
PARTIAL INDEXATION and PREFERENCE SHOCK	✓✓	✓✓	✓✓	✓✓

Notes: Observable variables are YGR_t, $INFL_t$ and INT_t. All three rank criteria come to the same conclusion, so we do not separately report the results. A ✓✓ indicates that all model parameters are theoretically identifiable, whereas $[\psi_\pi, \psi_y, \rho_R, \rho_\sigma]$ indicates that these parameters cannot be identified jointly.

according to the Bayesian learning rate indicator of Koop et al. (2013) for the FLEX-PRICE BASELINE, STEADY STATE BASELINE, FLEX-PRICE PREFERENCE SHOCK and FLEX-PRICE INDEXATION scenarios. We focus in particular on these scenarios as the non-identified model of An and Schorfheide (2007) corresponds to our FLEX-PRICE BASELINE scenario and the SW rule behaves similarly to the FLEX-PRICE specification. Our simulation and estimation exercise shows that the monetary policy parameters are weakly identifiable in the original (theoretically non-identified) model, since the rates at which the posterior precisions are updated are slower than the sample sizes. That is, the average posterior precision values in Table 9 tend towards zero instead of constant values, and, similarly, the convergence ratios in Table 10 do not tend towards the change in sample size of 3. Here it becomes evident that estimating non-identified models may also introduce problems in the estimation of other, actually identified model parameters. In particular, this is accompanied by many difficulties in the initialization of the proposal distribution for the MCMC algorithm as finding the mode and a positive definite Hessian at the mode is tedious, see section 3.2 on how we overcome this issue. Albeit, this is an inherent problem of many (even identified) DSGE models, lack of identification of some parameters aggravates this. If, however, the monetary policy authority reacts to output deviations from steady state all parameters, including the ones in the Taylor rule, are strongly identified. The same is true for the flex-price Taylor rule, when we introduce a partial inflation indexation scheme. A preference shock, on the other hand, leaves several parameters (ψ_π, ψ_y, ρ_R , σ_R, ρ_ζ and σ_ζ) weakly identified.

5.5. Summary

To sum up, in all our experiments we provide theoretical support (in terms of identification) for including both a PARTIAL INFLATION INDEXATION scheme as well as a PREFERENCE SHOCK into modern DSGE models. These features solve the theoretical lack of identification of the Taylor rule parameters independent of the output-gap specification. However, only PARTIAL INFLATION INDEXATION enhances the overall strength of identification of all model parameters, whereas the PREFERENCE SHOCK leaves several model parameters weakly identifiable (and estimable). Regarding the selection of observables, we find that some parameters, e.g. the average growth rate of technology, are only identifiable when introducing a specific measurement equation. This, of course, provides researchers another option to fine-tune the identifiability of their models (possibly with another eye to data availability). In this line of thought, we could have also introduced an additional equation that pins down target government spending g^*, otherwise it drops out from the linearized solution. Lastly, as mentioned in the beginning of the section, φ and ν are co-linear, as they jointly determine the slope of the New Keynesian Phillips curve. Our model variants are not able to separately identify these parameters, which is a common finding in linearized New Keynesian DSGE models, see e.g. Clarida et al. (1999), Ireland (2004) or Levin et al. (2003). We refer to Mutschler (2015) who shows that a higher-order approximation of the solution yields means to distinguish these parameters even in the BASELINE FLEX-PRICE scenario.

6. Implications for model building

Our results are relevant from a model building perspective, because it is crucial for macroeconomists to know what model features, frictions and shocks can coexist within models without redundancy. In our example models, we focus on four choices a researcher can make that matter for identification.

6.1. Choice of observables

First, both theoretical lack of and empirical weak identification are often due to an unfortunate choice of observables. In some cases, like the steady state parameters in our monetary model, this seems obvious. In other cases, like in our investment-adjustment costs model, one should use a specific observable variable (e.g. consumption) instead of other, commonly used ones (e.g. output). As the literature on the choice of observables is still very sparse (Canova et al., 2014; Guerron-Quintana, 2010; Martínez-García et al., 2012), we advocate (and show means) to do a brute-force sensitivity analysis before taking a model to actual data. In this line of thought, Kim (2003) already pointed out, that information on the relative price of investment can also solve the identification problem and hence, it is not surprising that current papers include this price in estimated DSGE models. However, there are trade-offs in terms of model-implied dynamics and empirical fit. For instance, Schmitt-Grohé and Uribe (2012) use the relative price of investment as an observable and find that their results regarding the macroeconomic effects of investment-specific shocks are in sharp contrast with the ones obtained by Justiniano et al. (2010) who do not include this variable in their estimation. Therefore, it is important, on the one hand, to establish which observables are theoretically favored in terms of identification, but, on the other hand, to use economic hindsight and model simulations to select the variables that best address the key issues one is interested in. Also, data availability and limitations need to be taken care of and one has to correctly transform empirical data to match the model variables, we refer to Pfeifer (2018) for excellent hands-on advice on this.

6.2. Functional specifications

Second, our finding that the investment-growth specification of intertemporal costs in the fashion of Christiano et al. (2005) is not subject to functional equivalence with multisectoral costs is useful, especially since this specification is now the benchmark in the quantitative DSGE literature. Accordingly, Christiano et al. (2011) compare the growth and level specifications of investment adjustment costs in their study of the government-spending multiplier at the zero lower bound. They find, while the growth specification implies a smaller response to investment (as it penalizes changes in investment directly), the dynamic responses of the other variables are similar, such that their main results are robust. Likewise, the monetary policy rule needs to be specified carefully and there is no consensus on the right functional specification. We follow up on the fact that economists employ quite different

Table 9
Average posterior precisions for monetary model.

PARAMETERS

T	r^A	π^A	γ^Q	τ	ν	ψ_π	ψ_y	ρ_R	ρ_g	ρ_z	$100\sigma_R$	$100\sigma_g$	$100\sigma_z$	ρ_ζ	$100\sigma_\zeta$	ι^p
BASELINE WITH FLEX-PRICE TAYLOR RULE (∗)																
100	0.073	0.045	0.495	0.081	11.648	0.503	0.396	12.218	3.386	35.239	36.472	4.059	9.896	–	–	–
300	0.042	0.023	0.233	0.044	10.966	0.212	0.103	8.774	8.550	25.330	31.041	5.302	8.838	–	–	–
900	0.024	0.013	0.151	0.025	6.222	0.119	0.045	6.610	9.153	26.105	33.604	5.510	8.101	–	–	–
2700	0.023	0.012	0.131	0.025	6.230	0.041	0.014	3.262	8.136	28.803	25.203	5.578	8.970	–	–	–
8100	0.021	0.012	0.126	0.025	6.745	0.015	0.005	1.360	8.067	28.419	13.812	5.578	9.217	–	–	–
BASELINE WITH STEADY STATE TAYLOR RULE																
100	0.075	0.025	0.523	0.070	9.020	0.693	1.831	11.527	4.703	39.077	36.586	2.246	9.015	–	–	–
300	0.039	0.009	0.221	0.047	7.927	0.322	0.719	8.912	11.735	21.345	34.834	2.254	5.373	–	–	–
900	0.022	0.007	0.143	0.020	4.564	0.415	1.519	10.241	11.832	23.252	42.893	2.608	4.985	–	–	–
2700	0.021	0.007	0.122	0.019	4.641	0.379	1.480	10.163	10.438	24.466	44.439	2.696	5.151	–	–	–
8100	0.021	0.007	0.127	0.019	5.292	0.436	1.780	10.079	9.613	22.966	41.976	2.630	5.109	–	–	–
PREFERENCE SHOCK WITH FLEX-PRICE TAYLOR RULE (∗)																
100	0.070	0.048	0.518	0.072	23.266	0.428	0.302	10.612	2.415	25.409	29.496	4.780	8.650	0.257	0.144	–
300	0.041	0.026	0.247	0.032	11.194	0.213	0.096	8.827	2.104	24.163	29.769	4.598	7.110	0.096	0.045	–
900	0.027	0.014	0.146	0.027	7.656	0.103	0.034	5.844	3.942	25.276	29.056	5.170	7.624	0.034	0.055	–
2700	0.028	0.014	0.153	0.017	7.115	0.042	0.013	3.033	4.256	22.480	23.579	2.697	5.685	0.023	0.016	–
8100	0.023	0.012	0.120	0.020	6.381	0.014	0.004	1.308	4.660	26.288	13.355	3.758	6.303	0.005	0.010	–
INDEXATION WITH FLEX-PRICE TAYLOR RULE (∗)																
100	0.074	0.041	0.494	0.077	12.615	0.539	0.325	12.077	3.268	32.418	35.518	4.083	7.383	–	–	1.083
300	0.042	0.022	0.234	0.051	11.921	0.314	0.125	8.045	8.271	23.608	28.648	5.308	6.991	–	–	0.632
900	0.025	0.012	0.153	0.027	7.086	0.311	0.084	6.286	9.069	23.826	32.700	5.472	5.346	–	–	0.580
2700	0.024	0.011	0.131	0.030	7.039	0.225	0.051	4.597	8.245	27.163	28.740	5.616	6.179	–	–	0.589
8100	0.021	0.010	0.119	0.030	7.507	0.203	0.042	3.772	8.074	26.849	25.771	5.590	6.179	–	–	0.584

Notes: Observable variables are YGR_t, $INFL_t$ and INT_t. A (∗) indicates cases where we used an advanced mode finding procedure, as outlined in section 3.2.

Table 10
Convergence ratios of posterior precisions for monetary model.

PARAMETERS																
dT	r^A	π^A	γ^Q	τ	ν	ψ_π	ψ_y	ρ_R	ρ_g	ρ_z	$100\sigma_R$	$100\sigma_g$	$100\sigma_z$	ρ_ζ	$100\sigma_\zeta$	ρ^p
BASELINE WITH FLEX-PRICE TAYLOR RULE (*)																
300/100	1.714	1.565	1.412	1.645	2.824	1.265	0.781	2.154	7.576	2.156	2.553	3.919	2.680	–	–	–
900/300	1.765	1.730	1.947	1.680	1.702	1.680	1.304	2.260	3.212	3.092	3.248	3.117	2.750	–	–	–
2700/900	2.824	2.726	2.599	2.973	3.004	1.036	0.920	1.481	2.667	3.310	2.250	3.037	3.322	–	–	–
8100/2700	2.781	2.857	2.876	3.003	3.248	1.091	1.045	1.250	2.974	2.960	1.644	3.000	3.082	–	–	–
BASELINE WITH STEADY STATE TAYLOR RULE																
300/100	1.564	1.087	1.264	2.004	2.636	1.393	1.178	2.319	7.485	1.639	2.856	3.011	1.788	–	–	–
900/300	1.725	2.462	1.950	1.277	1.727	3.865	6.341	3.448	3.025	3.268	3.694	3.471	2.784	–	–	–
2700/900	2.766	2.894	2.560	2.784	3.051	2.745	2.922	2.977	2.647	3.157	3.108	3.101	3.100	–	–	–
8100/2700	3.015	2.883	3.111	3.054	3.420	3.447	3.609	2.975	2.763	2.816	2.834	2.926	2.976	–	–	–
PREFERENCE SHOCK WITH FLEX-PRICE TAYLOR RULE (*)																
300/100	1.759	1.645	1.429	1.338	1.443	1.490	0.952	2.495	2.613	2.853	3.028	2.886	2.466	1.120	0.934	–
900/300	1.990	1.574	1.776	2.550	2.052	1.448	1.070	1.986	5.620	3.138	2.928	3.374	3.217	1.076	3.697	–
2700/900	3.106	3.122	3.140	1.906	2.788	1.238	1.171	1.557	3.239	2.668	2.434	1.565	2.237	2.036	0.853	–
8100/2700	2.418	2.482	2.347	3.476	2.691	0.990	0.960	1.294	3.285	3.508	1.699	4.180	3.326	0.703	1.884	–
INDEXATION WITH FLEX-PRICE TAYLOR RULE (*)																
300/100	1.707	1.569	1.425	1.977	2.835	1.751	1.149	1.999	7.592	2.185	2.420	3.901	2.841	–	–	1.751
900/300	1.802	1.740	1.952	1.581	1.783	2.964	2.019	2.344	3.289	3.028	3.424	3.092	2.294	–	–	2.752
2700/900	2.797	2.676	2.575	3.338	2.980	2.170	1.833	2.194	2.728	3.420	2.637	3.079	3.468	–	–	3.049
8100/2700	2.610	2.688	2.721	2.990	3.199	2.712	2.450	2.462	2.938	2.965	2.690	2.986	3.000	–	–	2.971

Notes: Observable variables are YGR_t, $INFL_t$ and INT_t. A (*) indicates cases where we used an advanced mode finding procedure, as outlined in section 3.2.

concepts and definitions for the output-gap (Kiley, 2013), and show how this matters for identification. This is also consistent with Hirose and Naganuma (2010)'s findings, who argue that the estimated output gap is sensitive to the specification of monetary policy rules.

6.3. Model features

Third, adding model features provides a researcher with more flexibility in functional specifications. Accordingly, our findings show that by adding a cost on capital utilization one is able to identify models with both multisectoral and intertemporal costs. A recent example of this is Moura (2018) who is able to estimate both types of costs to study investment price rigidities in a multisectoral DSGE model. Likewise, partial inflation indexation identifies our monetary model, independent of the concrete specification of the Taylor rule, and even enhances the strength of identification of all model parameters. This finding mirrors the fact that history dependence of inflation in the New Keynesian Phillips curve improves the fit of an otherwise forward-looking model, as emphasized by Smets and Wouters (2003, 2007).

6.4. Choice of shocks

Fourth, introducing additional innovations, like structural shocks to investment-specific technological change or to the preference discount rate, plays an important role not only for the model dynamics, but also in terms of theoretical and empirical parameter identifiability. This finding is of a more general nature and needs some discussion.

On the one hand, shocks introduce wedges and different internal dynamics to the structure of a model. For example, additional shocks add additive components to the variance decomposition. If a previously unidentified parameter influences this additional component, there is a way to identify this parameter from the second moments or spectral properties of data. Of course, it is important that the additional component is not simply a linear combination of other components of already included shocks. Similar to our findings, Martínez-García and Wynne (2014, p. 166) argue that a productivity shock has the potential to introduce differences in models that can be exploited to tell them apart. Equivalently, Canova et al. (2014, p. 435) and Guerron-Quintana (2010) provide illustrative three equation models to show analytically that different shocks carry different information for parameter identification. Our example models, however, are not analytically but empirically motivated. The results in our paper, therefore, resemble the findings that (1) general equilibrium models perform poorly when explaining investment dynamics without heavily relying on investment specific shocks (Justiniano et al., 2011; Kamber et al., 2016), (2) the variability of inflation is to a large extent determined by preference shocks (Smets and Wouters, 2003) and (3) shocks to the preference discount rate play an important role in getting the interest rate fall to zero (Christiano et al., 2011).

On the other hand, in practice, the number of shocks limits the number of observable variables one can choose. Applied researchers may be tempted to add non-structural shocks such as measurement errors, but we advise not to do so because of two reasons. First, the results on identification may be counter-productive as indicated by e.g. Martínez–García et al. (2012). In our example models, we experimented with measurement errors and found no significant benefit of adding these. Second, even though measurement errors in the observation equation are a necessary requirement for some estimation procedures and filtering techniques, they potentially affect the accuracy of parameter estimates (Atkinson et al., 2019).

7. Conclusion

We strongly recommend that researchers treat parameter identification as a model property, i.e. from a model building perspective. A wise choice on observables or slight and subtle changes and fine-tuning of model assumptions, functional specifications, or structural shocks have an impact on both theoretical (yes/no) identification properties as well as on the strength of identification. In this regard, we side with Adolfson et al. (2019) who argue that "lack of identification should neither be ignored nor be assumed to affect all DSGE models, [...] identification problems can be readily assessed on a case-by-case basis". We extend their approach by using different diagnostic tools for theoretical as well as empirical identification properties and also show means to dissolve the identification failures. Moreover, our paper also has a computational contribution as our research feeds into and extends Dynare's *identification toolbox*. In particular, we provide means to analyze the criteria of Komunjer and Ng (2011) and Qu and Tkachenko (2012) by using analytical (instead of numerical) derivatives to compute the relevant Jacobians. Lastly, even though our example models are empirically motivated, they are still of small scale and easy to replicate and extend. They should be useful for both applied and theoretical macroeconomists as well as for teaching purposes.

Declaration of competing interest

None.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.econmod.2019.09.039.

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