A note on functional equivalence between intertemporal and multisectoral investment adjustment costs

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Abstract

Kim (2003, JEDC 27, pp. 533-549) shows functional equivalence between intertemporal and multisectoral investment adjustments costs in a log-linearized RBC model. We provide two strategies to solve this equivalence. First, the equivalence does not hold when intertemporal adjustment costs are specified in growth rates rather than in levels. Second, the level specification can be identified with a second-order approximation of the model solution. We estimate the quadratic approximation using two extended Kalman filters within a Bayesian framework. Our estimation results confirm that both parameters are estimable in finite samples. Moreover, we provide further evidence on the stabilizing effect of pruning on the estimation algorithm.

Keywords: identification, quadratic approximation, pruning, nonlinear DSGE, investment adjustment costs

JEL: C13, C51, E22, E32, O41

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1. Introduction

Current macroeconomic models commonly incorporate investment adjustment costs in order to account for the fact that investment in capital is costly. Intuitively, a firm can neither instantly change its capital stock nor immediately produce e.g. automobiles instead of books without some costs of adjustments, as it takes time and resources to change the composition of goods produced. As Kim (2003a, p. 533f) notes:

> Two types of adjustment costs specifications coexist in the macroeconomics literature on investment. One type specifies intertemporal adjustment costs in terms of a nonlinear substitution between capital and investment in capital accumulation, as in Lucas & Prescott (1971), Hayashi (1982), and Abel & Blanchard (1983). The other specification captures multisectoral adjustment costs by incorporating a nonlinear transformation between consumption and investment, which is used by Sims (1989), Vallès (1997), and many other papers adopting multisector models.

Multisectoral investment adjustment costs, on the one hand, provide models with a strong propagation mechanism and can successfully explain co-movements between sectors without relying upon any extra features or frictions, see e.g. Greenwood et al. (2000) and Huffman & Wynne (1999). Intertemporal investment adjustment costs, on the other hand, are most commonly used in state-of-the-art dynamic stochastic general equilibrium (DSGE) models as they untangle the linkage to the marginal product of capital, therefore, explaining the acyclic behavior of the real interest rate. The papers mentioned in the quote propose a specification which is based on the first derivative of capital or, in other words, on the current level of investment. It finds use in current DSGE models developed by e.g. the Czech National Bank (Andrle et al., 2009) or the Council for Budget Responsibility for Slovakia (Mucka & Horvath, 2015). However, due to the popularity of models in the fashion of Christiano et al. (2005) and Smets & Wouters (2003), it is now common to use a specification which depends on the current growth rate of investment. Even though Christiano et al. (2005) note that this specification successfully generates persistent, hump-shaped responses of aggregate investment and output to monetary policy shocks, Groth & Khan (2010) find no empirical evidence for this kind of specification. They, however, focus their empirical analysis on single equations without taking the cross-restrictions from a full model into account. Moreover, the variables used for the marginal product of capital are likely misspecified, since these typically underestimate the nonlinearities due to factor complementarities and time-varying markups (Linnemann, 2016).

Multisectoral and intertemporal investment adjustment costs provide meaningful model dynamics in different strands of literature. The theoretical relationship between macroeconomic (in)stability and investment adjustment costs has been studied by (among others) Chin et al. (2012), Kim (2003b) and Herrendorf & Valentinyi (2003). The influence of investment adjustment costs on news-driven cycles and co-movements has produced a large literature strand both for intertemporal as well as multisectoral specifications: Guo et al. (2015) and Jaimovich & Rebelo (2009) use intertemporal costs to generate news-driven business cycles, whereas Beaudry & Portier (2007) argue that multisectoral costs can support positive co-movements between consumption, investment and employment due to changes in expectations in a perfect market environment with variable labor supply. Dupor & Mehkari (2014) and Qureshi (2014) support the evidence that multisectoral investment adjustment costs lead to positive sectoral and aggregate co-movement in response to news shocks. The regained interest in using a multisectoral specification is also evident in the residential investment literature, see Kydland et al. (2016) and Garriga et al. (2017). Lastly, there is some evidence that financial frictions and investment adjustment costs yield almost observational equivalent models, see Bayer (2008), Casalin & Dia (2014) and Ikeda (2011).

The combination of both intertemporal and multisectoral investment adjustment costs is, however, not (or at best rather sparsely) used in macroeconomic models.[1] This is mainly due to the functional equivalence result of Kim (2003a):

> [W]hen a model already has a free parameter for intertemporal adjustment costs, adding another parameter for multisectoral adjustment costs does not enrich the model dynamics (Kim, 2003a, p. 534).

[1] Moura (2015) is a recent exception who uses both types of costs to study investment price rigidities in a multisectoral DSGE model. It is argued, however, without a formal proof, that the inclusion of intersectoral frictions solves the functional equivalence result of Kim (2003a).

This means that, economically speaking, an increase in multisectoral adjustment costs can be equivalent to a decrease in intertemporal adjustment costs. From an identification point of view this relates to two parameters being collinear, and thus not separately identifiable. Specifically, in the log-linearized (first-order approximated) solution of Kim (2003a)'s RBC model the parameters governing intertemporal and multisectoral investment adjustment costs enter as a ratio and are hence not separately identifiable no matter what estimation method one uses.

The goal of our paper is to provide two strategies – a theoretical and an econometric one – to solve the functional equivalence and therefore provide macroeconomic modelers with means to include both types of investment adjustment costs. To this end, we formally show that the functional equivalence result does not hold for the growth rate specification of intertemporal investment adjustment costs. Likewise we use insights of Mutschler (2015) who has shown that for the level specification the functional equivalence is due to the linear approximation of the solution of the model, a quadratic approximation provides restrictions on the moments to identify both parameters separately. Our econometric contribution begins here, as we estimate the quadratic approximation to demonstrate that the parameters are also estimable in finite samples. To this end, we simulate data for different parameter values and estimate the model with Bayesian methods. As second-order approximated models tend to generate exploding simulated time paths, we analyze the effect of pruning on our estimation results.[2] That is, we replace second-order terms in the solution by products of the linearized solution and simulate data both from the pruned as well as unpruned quadratic approximation. We use two different (extended) Kalman filters, namely the *Central Difference Kalman Filter* (Andreasen, 2011) and the *Quadratic Kalman Filter* (Ivashchenko, 2014), to evaluate the quadratic likelihood. We specifically account for the effect of pruning and no pruning in the updating step of both filters similar to Kollmann (2015). We find that pruning inside the filter has a stabilizing effect on the estimation of nonlinear DSGE models independent of (possible) explosiveness in the true data-generating-process.

[2]For a thorough exposition of pruning see Andreasen et al. (2016) and Kim et al. (2008).

2. The Kim (2003) model

The Kim (2003a) model builds upon the canonical neoclassical growth model (see for example Schmitt-Grohé & Uribe (2004)), however, augmenting it with two kinds of investment adjustment costs. First, intertemporal adjustment costs are introduced into the capital accumulation equation governed by a parameter ϕ. We will consider two specifications. The level specification involves a nonlinear substitution between capital k_t and investment i_t:

$$k_t = \left[\delta\left(\frac{i_t}{\delta}\right)^{1-\phi} + (1-\delta)\left(k_{t-1}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{1}$$

with δ denoting the depreciation rate. The growth rate specification involves costs in terms of investment changes between periods. That is, the cost is increasing in the change in investment:

$$k_t = (1-\delta)k_{t-1} + i_t\left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) \tag{2}$$

where it is usually assumed that $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$. As functional form, we set $S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$. Note that $\phi = 0$ implies the usual linear capital accumulation specification in both cases.

Second, we introduce multisectoral adjustment costs into the national budget constraint given a parameter θ, which are captured by a nonlinear transformation between consumption c_t and investment i_t:

$$a_{t-1}k_{t-1}^{\alpha} = \left[(1-s)\left(\frac{c_t}{1-s}\right)^{1+\theta} + s\left(\frac{i_t}{s}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}} \tag{3}$$

with a_t denoting the level of technology. The steady-state savings rate, $s = \frac{c}{y} = \frac{\beta\delta\alpha}{1-\beta+\delta\beta}$, consists of the depreciation rate δ, the discount factor β and the share of capital in production α. Similar to Huffman & Wynne (1999) we focus on $\theta > 0$, i.e. a *reverse CES* technology, in order for the production possibilities set to be convex. Thus, it becomes more difficult to transfer goods previously used in the consumption sector one-to-one into the investment-good sector. Note that for $\theta = 0$ the transformation is linear.

The representative agent maximizes $E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$ subject to the budget constraint (3) and the capital accumulation equation (1) or (2). Denote λ_t and $\lambda_t Q_t$ as the corresponding Lagrangian multipliers, then the first-order conditions for the level specification are:

$$\begin{aligned}
\lambda_t &= \frac{1}{c_t}\left(\frac{c_t}{(1-s)a_{t-1}k_{t-1}^{\alpha}}\right)^{-\theta}, \\
Q_t &= \left(\frac{i_t}{s a_{t-1}k_{t-1}^{\alpha}}\right)^{\theta}\left(\frac{i_t}{\delta k_t}\right)^{\phi}, \\
\lambda_t Q_t &= \beta\lambda_{t+1}\left\{\alpha a_t k_t^{\alpha-1} + Q_{t+1}(1-\delta)\left(\frac{k_{t+1}}{k_t}\right)^{\phi}\right\},
\end{aligned}$$

and for the growth rate specification:

$$\begin{aligned}
\lambda_t &= \frac{1}{c_t}\left(\frac{c_t}{(1-s)a_{t-1}k_{t-1}^{\alpha}}\right)^{-\theta}, \\
Q_t\left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) &= Q_t i_t S'\left(\frac{i_t}{i_{t-1}}\right) + \beta Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t} i_{t+1} S'\left(\frac{i_{t+1}}{i_t}\right) + \left(\frac{i_t}{s a_{t-1}k_{t-1}^{\alpha}}\right)^{\theta}, \\
\lambda_t Q_t &= \beta\lambda_{t+1}\left(\alpha a_t k_t^{\alpha-1} + Q_{t+1}(1-\delta)\right).
\end{aligned}$$

with $S'(\cdot) = \partial S'(\cdot)/\partial i_t$. Note that for $\phi = \theta = 0$ both simplify to the canonical Euler equation. To close the model, we assume that technology evolves according to a stable AR(1) process $log(a_t) = \rho_a log(a_{t-1}) + \varepsilon_{a,t}$ with ρ_a measuring persistence and $\varepsilon_{a,t} \sim iid(0, \sigma_a^2)$. The steady state of the model is given by

$$a = 1,\ Q = 1,\ k = \left(\frac{\delta}{sa}\right)^{\frac{1}{\alpha-1}},\ i = \delta k, c = (1-s)\left[\frac{(\alpha k^{\alpha})^{1+\theta} - s\left(\frac{i}{s}\right)^{1+\theta}}{1-s}\right]^{\frac{1}{1+\theta}}.$$

There are two exogenous (k_t and a_t) and no endogenous state variables. The control variables, c_t and i_t, are both assumed to be observable and measured with errors $\varepsilon_{c,t} \sim iid(0, \sigma_c^2)$ and $\varepsilon_{i,t} \sim iid(0, \sigma_i^2)$. For our econometric strategy, we fix $\beta = 0.99$ and $\delta = 0.0125$ at standard values. Our prior beliefs and bounds for the remaining parameters are summarized in table 1.

3. Identification analysis

In the original paper Kim (2003a) log-linearizes the model around the non-stochastic steady-state and shows analytically that there is observational equivalence between multisectoral and intertemporal (based on levels) investment adjustment costs: θ and ϕ enter as a ratio $\frac{\phi+\theta}{1+\theta}$ into the log-linearized solution; hence, they are not distinguishable. This can also be verified via a formal local identification analysis for log-linearized models as in Iskrev (2010) and Qu & Tkachenko (2012). Intuitively, we want that no two parameter vectors yield the same moments of our observables. In a nutshell, Iskrev (2010)'s approach checks whether the mean, variance and autocovariogram of the observables are sensitive to changes of the structural parameters, whereas Qu & Tkachenko (2012)'s approach focuses on the mean and spectrum of the observables. These changes are measured by Jacobian matrices which are required to have full rank. If we get rank shortages we can analyze the null space of the Jacobians to pinpoint the problematic parameters. Parameters that do not enter the solution will correspond to columns of zeros, whereas for parameters like θ and ϕ we get linearly dependent columns.

Columns two and three of table 2 summarize the ranks for the log-linearized solution of the model with level specification. For all used tolerance levels (as we compute ranks via a singular value decomposition) the rank is short by one. Analyzing the nullspace indicates that indeed one has to fix either θ or ϕ to identify the model. Mutschler (2015) establishes the same criteria for higher-order approximations. Columns four and five in table 2 display the corresponding ranks for the quadratic approximation. We get full rank, i.e. full local identification at the prior mean, in all cases.

The same is true, when we analyze the log-linearized solution of the model with the growth rate specification for the intertemporal adjustment costs. Columns six and seven of table 2 confirm that all parameters are identifiable, so there is no need to consider a quadratic approximation for this specification. This result is – as far as we know – new to the literature and provides further support to choose the growth rate specification for intertemporal investment adjustment costs in DSGE models.

4. Monte-Carlo study

4.1. Solution method and data-generating-process

The exact solution of our nonlinear model is given by a set of decision rules g and h for state variables $x_t = (k_{t-1}, a_{t-1})'$ and control variables $y_t = (c_t, i_t, Q_t)'$, that is:

$$y_{t+1} = g(x_t, u_{t+1}, \sigma), \qquad x_{t+1} = h(x_t, u_{t+1}, \sigma).$$

and $u_t = (\varepsilon_{a,t}, \varepsilon_{c,t}, \varepsilon_{i,t})$. We introduce the perturbation parameter σ and approximate the functions g and h using a quadratic Taylor approximation around the non-stochastic steady-state ($\sigma = 0$) following e.g. Schmitt-Grohé & Uribe (2004). Therefore our first data-generating-process (DGP 1) is given by:

DGP 1 (Unpruned)**.**

$$\begin{aligned}
\hat{x}_{t+1} &= h_x \hat{x}_t + h_u u_{t+1} + \frac{1}{2} H_{xx} \left(\hat{x}_t \otimes \hat{x}_t\right) + \frac{1}{2} H_{uu} \left(u_{t+1} \otimes u_{t+1}\right) \\
&\quad + \frac{1}{2} H_{xu} \left(\hat{x}_t \otimes u_{t+1}\right) + \frac{1}{2} H_{ux} \left(u_{t+1} \otimes \hat{x}_t\right) + \frac{1}{2} h_{\sigma\sigma} \sigma^2, \\
\hat{y}_{t+1} &= g_x \hat{x}_t + g_u u_{t+1} + \frac{1}{2} G_{xx} \left(\hat{x}_t \otimes \hat{x}_t\right) + \frac{1}{2} G_{uu} \left(u_{t+1} \otimes u_{t+1}\right) \\
&\quad + \frac{1}{2} G_{xu} \left(\hat{x}_t \otimes u_{t+1}\right) + \frac{1}{2} G_{ux} \left(u_{t+1} \otimes \hat{x}_t\right) + \frac{1}{2} g_{\sigma\sigma} \sigma^2.
\end{aligned}$$

A hat denotes deviations from steady-state, e.g. $\hat{y}_t = y_t - \bar{y}$. h_x and g_x denote the solution matrices of the first-order approximation, H_{xx} is a 2×2^2 matrix containing all second-order terms for the i-th state variable in the i-th row, whereas G_{xx} is a 3×2^2 matrix containing all second-order terms for the i-th control variable in the i-th row. H_{xu}, H_{ux}, G_{xu} and G_{ux} are accordingly shaped for the cross terms of states and shocks, and H_{uu} and G_{uu} contain the second-order terms for the product of shocks.

Various simulation studies show that Taylor approximations of an order higher than one may generate explosive time paths, even though the first-order approximation is stable. This is due to artificial fixed points of the approximation, see Kim et al. (2008, p. 3408) for a univariate example. Thus, the model may be neither stationary nor imply an ergodic probability distribution, both of which assumptions are essential for identification and estimation. Thus, Kim et al. (2008) propose the pruning scheme, in which one omits terms from the policy functions that have higher-order effects than the approximation order.[3] For instance, given a second-order approximation, we decompose the state vector into first-order ($\hat{x}_t^f$) and second-order ($\hat{x}_t^s$) effects ($\hat{x}_t = \hat{x}_t^f + \hat{x}_t^s$), and set up the law of motions for these variables, preserving only effects up to second-order (see the technical appendix of Andreasen et al. (2016) for details). Our second data-generating-process (DGP 2) is hence given by:

DGP 2 (Pruned)**.**

$$
\begin{aligned}
\hat{x}_{t+1}^f &= h_x \hat{x}_t^f + h_u u_{t+1}, \\
\hat{x}_{t+1}^s &= h_x \hat{x}_t^s + \frac{1}{2} H_{xx} \left(\hat{x}_t^f \otimes \hat{x}_t^f \right) + \frac{1}{2} H_{uu} \left(u_{t+1} \otimes u_{t+1} \right) \\
&\quad + \frac{1}{2} H_{xu} \left(\hat{x}_t^f \otimes u_{t+1} \right) + \frac{1}{2} H_{ux} \left(u_{t+1} \otimes \hat{x}_t^f \right) + \frac{1}{2} h_{\sigma\sigma} \sigma^2, \\
\hat{y}_{t+1} &= g_x (\hat{x}_t^f + \hat{x}_t^s) + g_u u_{t+1} + \frac{1}{2} G_{xx} \left(\hat{x}_t^f \otimes \hat{x}_t^f \right) + \frac{1}{2} G_{uu} \left(u_{t+1} \otimes u_{t+1} \right) \\
&\quad + \frac{1}{2} G_{xu} \left(\hat{x}_t^f \otimes u_{t+1} \right) + \frac{1}{2} G_{ux} \left(u_{t+1} \otimes \hat{x}_t^f \right) + \frac{1}{2} g_{\sigma\sigma} \sigma^2.
\end{aligned}
$$

Thus, terms containing $\hat{x}_t^f \otimes \hat{x}_t^s$ and $\hat{x}_t^s \otimes \hat{x}_t^s$ are omitted (i.e. pruned), since they reflect third-order and fourth-order effects which are higher than the approximation order. Also, there are no second-order effects in u_{t+1}.

For our Monte-Carlo study we draw 50 values from the prior domain given in table 1 that yield a determinate solution. For each of these draws we simulate paths of the observable variables of $T = 100$ using both the (possibly) explosive DGP 1 and stable DGP 2.

4.2. Estimation method

Due to the quadratic approximation we are faced with nonlinearities such that we cannot use the standard Kalman filter to evaluate the likelihood. There is, however, a growing literature on estimating nonlinear solutions to DSGE models, including Quasi-Maximum-Likelihood estimation (Andreasen, 2011; Ivashchenko, 2014; Kollmann, 2015)

[3] This may seem an ad hoc procedure, but pruning can also be founded theoretically as a Taylor expansion in the perturbation parameter (Lombardo & Uhlig, 2014) or on an infinite moving average representation (Lan & Meyer-Gohde, 2013).

and Bayesian Sequential Monte Carlo methods (An & Schorfheide, 2007; Fernández-Villaverde & Rubio-Ramírez, 2007; Herbst & Schorfheide, 2014). We follow this literature and estimate our model parameters with Bayesian methods. Our MCMC algorithm requires a filtering step to evaluate the likelihood, for which we use four different approaches: (1) Central Difference Kalman Filter (CDKF from now on), (2) Central Difference Kalman Filter taking specifically the pruned solution into account in the updating step (CDKFP from now on), (3) Quadratic Kalman Filter (QKF from now on), and (4) Quadratic Kalman Filter taking specifically the pruned solution into account in the updating step (QKFP from now on). Therefore we extend results of Andreasen (2011) and Ivashchenko (2014) and tune the filters in the fashion of Kollmann (2015) to account for the stabilizing effect of pruning. The obtained likelihood is, however, often badly shaped, multimodal and has discontinuities. The evaluation of first-order and second-order derivatives is intractable and gradient based optimization methods perform quite poorly. Therefore, we use an optimization routine that is based on simulations, namely, the evolutionary algorithm CMA-ES, see Andreasen (2010) for an application to DSGE models. The rest of the Bayesian framework is standard, as we use a random walk Metropolis-Hastings algorithm as in Schorfheide (2000) and DYNARE. That is, we run two chains, each with 15000 draws, which are initialized at the posterior mode and using the inverse hessian for the initial proposal covariance matrix. We use DYNARE to simulate, solve and estimate the model with and without pruning. We specifically implement procedures for the different Kalman filters to work within DYNARE.[4]

5. Estimation results

We estimate the parameters of the model (with the level specification of intertemporal costs) using each of the four different Kalman filters within a Bayesian framework for each of the two DGPs. First, we present the bias (posterior mean - true value) and standard deviation of the posterior draws for the well-identified parameter α in table 3. Here it is evident that all filters are perfectly capable to pinpoint α precisely, as the biases and posterior standard deviations are very small. For the explosive DGP 1 the

[4]The procedures are available on request.

QKF and QKFP are slightly better in terms of smaller bias than the CDKF and CDKFP. All filters perform slightly better for the stable DGP 2 compared to DGP 1. Regarding stability we find that including pruning in the updating step of the filters implies less unreliable and more efficient estimates regardless of the DGP.

We now turn to the *problematic* parameters θ and ϕ. Tables 4 and 5 depict the bias (posterior mean - true value) and standard deviations of the posterior draws for θ and ϕ, respectively. There is apparently more learning from data for ϕ than for θ. The bias and standard errors, however, are in both cases not negligible, but rather large, indicating that these parameters are at best weakly identified. This is not surprising as our sample size is very small with just 100 data points and we limit ourselves with only a small number of draws in the MCMC chains. Accordingly, we experienced computational errors and instabilities in several MC runs due to badly shaped inverse Hessians, low acceptance ratios and non-convergence of the chains, such that one would need to individually tune and optimize the settings for each Monte-Carlo run and filter to get more reliable estimation results. This is evident as in some instances we get a standard error of 0.000, which does not indicate very high precision, but that something went wrong in the estimation. Likewise for very large biases we suspect that the sampler did not travel through the whole posterior domain but was stuck at a local mode. A thorough estimation exercise with mode-finding from different starting points and inspection of slices through the likelihood is beyond the scope of our analysis, as we only want to demonstrate that both parameters are estimable. We do, however, assess that these problems are much more frequent when we do not account for pruning in the updating step. In particular, using the QKFP yields, in our experience, the most reliable results. Nevertheless, for each MC run at least one of the filters provides estimates within a reasonable credibility set, which is the goal of our estimation strategy.

To sum up, our Monte-Carlo results confirm that all approaches are able to extract information to provide meaningful estimates for both intertemporal as well as multisectoral adjustment cost parameters separately. Regarding stability we find that accounting for pruning in the updating step of the filters eases the estimation regardless whether our data is generated by the explosive DGP 1 or the stable DGP 2. Lastly, we comment on estimation speed: the computation of the posterior with 2 chains and 15000 draws each took about 40 minutes, whereas the computation of the mode using the CMA-ES took about 5 minutes on a standard desktop computer.

6. Conclusion

Econometrically, we show that both the *Central Difference Kalman Filter* as well as the *Quadratic Kalman Filter* are very powerful tools to estimate pruned as well as unpruned nonlinear DSGE models, even when the likelihood is badly shaped and we are faced with weakly identified parameters. We are able to estimate structural parameters that are unidentifiable under the log-linearized model; thus, confirming the findings of Mutschler (2015) in finite samples. In this sense, our paper is similar to An & Schorfheide (2007) who likewise estimate a small-scale DSGE model solved by a quadratic approximation. They, however, rely on the (time-consuming) particle filter to evaluate the likelihood and do not discuss the effect of pruning. The result is the same: Estimating the quadratic approximation of a DSGE model provides means to extract more information on the structural parameters from data. Similarly, Kim & Kwok (2007) analyze the Exchange Rate Dynamics Redux model of Obstfeld & Rogoff (1995) and show that a first-order equivalence does not extend to a second-order equivalence. Our paper provides means to execute a fully fledged estimation exercise of this model as well, which opens an area for further research.

Economically, we show that specifying intertemporal adjustment costs that are based on the growth rate of investment is another way to solve the functional equivalence between intertemporal and multisectoral costs even in log-linearized models. As this type is most commonly used in state-of-the-art DSGE models, it will be interesting to analyze the enriched model dynamics and propagation mechanisms by introducing multisectoral adjustment costs into these models as well. In this line of thought, our results are not limited to investment adjustment costs. Similar specifications are used to model imperfect labor mobility between the consumption-sector and the investment-sector, see e.g. Nadeau (2009). We leave these topics for further research.

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Table 1: Priors and bounds

	Prior specification			*Bounds*	
Parameter	Density	Para (1)	Para (2)	Lower	Upper
α	Gamma	0.60	0.30	1e-5	1
θ	Normal	1.00	0.50	-5	5
ρ_a	Beta	0.50	0.20	1e-5	0.99999
ϕ	Normal	2.00	0.50	-5	5
σ_a	Inverse Gamma	0.50	4.00	1e-8	5
σ_c	Inverse Gamma	0.50	4.00	1e-8	5
σ_i	Inverse Gamma	0.50	4.00	1e-8	5
β	Fixed	0.99	-	-	-
δ	Fixed	0.0125	-	-	-

Notes: Para (1) and (2) list on the one hand the means and the standard deviations for the Beta, Gamma and Normal distributions, and on the other hand s and v for the Inverse Gamma distribution, where $\wp_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$.

Table 2: Identification analysis of the Kim (2003) model

	Level Specification				**Growth Rate Specification**	
	Log-linearized		*Quadratic*		*Log-linearized*	
tol	Iskrev	Qu/Tkachenko	Iskrev	Qu/Tkachenko	Iskrev	Qu/Tkachenko
1e-07	6	6	7	7	7	7
1e-08	6	6	7	7	7	7
1e-09	6	6	7	7	7	7
1e-10	6	6	7	7	7	7
1e-11	6	6	7	7	7	7
1e-12	6	6	7	7	7	7
1e-13	6	6	7	7	7	7
robust	6	6	7	7	7	7
required	7	7	7	7	7	7

Notes: Ranks of Jacobians computed with analytical derivatives for different tolerance levels *tol*. Iskrev (2010)'s Jacobian is based on the derivative of the mean and autocovariogram computed with 30 lags, whereas Qu & Tkachenko (2012)'s Jacobian is based on the derivative of the mean and spectrum computed with 10000 subintervals. Implementation for first-order (*log-linearized*) and second-order (*quadratic*) approximated solutions of DSGE models is taken from Mutschler (2015).

Table 3: Bias for α

	DGP UNPRUNED				DGP PRUNED			
MC run	CDKF	CDKFP	QKF	QKFP	CDKF	CDKFP	QKF	QKFP
Average	0.011 (0.002)	−0.012 (0.003)	0.005 (0.003)	−0.006 (0.002)	0.001 (0.002)	−0.009 (0.003)	−0.007 (0.003)	−0.004 (0.002)
1	0.192 (0.008)	0.014 (0.006)	0.013 (0.006)	0.014 (0.006)	$0.110^{\dagger}$ (0.000)	0.034 (0.007)	0.059 (0.010)	0.060 (0.006)
2	−0.003 (0.000)	−0.004 (0.000)	$0.000^{\dagger}$ (0.000)	−0.003 (0.000)	−0.003 (0.000)	−0.004 (0.000)	$0.000^{\dagger}$ (0.000)	−0.007 (0.000)
3	−0.005 (0.001)	−0.005 (0.000)	0.007 (0.003)	−0.008 (0.000)	−0.004 (0.001)	−0.007 (0.001)	$0.000^{\dagger}$ (0.000)	−0.007 (0.000)
4	0.003 (0.003)	0.001 (0.002)	0.212 (0.001)	−0.000 (0.002)	−0.004 (0.004)	−0.004 (0.002)	0.001 (0.002)	0.000 (0.002)
5	−0.010 (0.001)	−0.007 (0.000)	−0.005 (0.000)	−0.009 (0.000)	−0.008 (0.000)	$-0.000^{\dagger}$ (0.000)	0.003 (0.001)	−0.007 (0.000)
6	0.016 (0.008)	0.003 (0.003)	−0.001 (0.004)	0.006 (0.004)	0.012 (0.002)	−0.014 (0.008)	0.006 (0.002)	0.000 (0.004)
7	−0.015 (0.001)	−0.016 (0.000)	$0.000^{\dagger}$ (0.000)	−0.015 (0.000)	−0.007 (0.001)	−0.005 (0.000)	$-0.000^{\dagger}$ (0.000)	−0.008 (0.000)
8	−0.005 (0.003)	−0.021 (0.002)	−0.017 (0.006)	−0.002 (0.000)	−0.006 (0.003)	−0.006 (0.000)	−0.048 (0.001)	0.002 (0.001)
9	−0.009 (0.001)	−0.009 (0.001)	$0.000^{\dagger}$ (0.000)	−0.006 (0.000)	−0.007 (0.001)	−0.004 (0.001)	−0.011 (0.003)	−0.010 (0.002)
10	0.131 (0.009)	−0.033 (0.005)	−0.039 (0.006)	−0.025 (0.013)	0.110 (0.007)	−0.005 (0.006)	0.005 (0.002)	−0.010 (0.006)
11	−0.009 (0.001)	−0.008 (0.000)	−0.005 (0.000)	−0.008 (0.000)	−0.007 (0.001)	−0.016 (0.000)	−0.010 (0.000)	0.004 (0.000)
12	−0.001 (0.002)	0.003 (0.002)	0.003 (0.002)	0.001 (0.003)	−0.002 (0.003)	0.002 (0.002)	0.004 (0.002)	0.001 (0.002)
13	0.020 (0.007)	−0.016 (0.001)	−0.008 (0.002)	−0.011 (0.003)	−0.013 (0.006)	−0.018 (0.002)	−0.010 (0.002)	−0.016 (0.003)
14	0.010 (0.003)	−0.012 (0.003)	−0.023 (0.003)	−0.015 (0.004)	−0.008 (0.003)	−0.009 (0.002)	−0.008 (0.003)	−0.006 (0.003)
15	0.005 (0.002)	0.001 (0.001)	0.030 (0.001)	−0.005 (0.001)	0.001 (0.000)	−0.007 (0.001)	0.024 (0.006)	−0.014 (0.001)
16	−0.009 (0.002)	−0.012 (0.001)	−0.010 (0.000)	−0.011 (0.003)	−0.010 (0.001)	−0.011 (0.001)	−0.001 (0.000)	−0.008 (0.000)
17	−0.007 (0.004)	−0.005 (0.002)	−0.009 (0.002)	−0.006 (0.003)	−0.005 (0.002)	0.003 (0.001)	−0.003 (0.002)	−0.016 (0.001)
18	−0.011 (0.002)	−0.021 (0.000)	−0.001 (0.000)	−0.011 (0.001)	−0.021 (0.002)	−0.007 (0.000)	−0.022 (0.000)	−0.026 (0.001)
19	0.029 (0.004)	−0.011 (0.005)	$0.362^{\dagger}$ (0.000)	0.003 (0.004)	0.002 (0.001)	−0.004 (0.003)	0.005 (0.004)	0.005 (0.004)

20	−0.002 (0.003)	−0.012 (0.002)	−0.006 (0.002)	−0.011 (0.002)	0.023 (0.006)	−0.006 (0.003)	−0.007 (0.003)	−0.005 (0.004)
21	−0.001 (0.003)	−0.008 (0.001)	−0.005 (0.001)	−0.007 (0.001)	−0.001 (0.003)	−0.007 (0.001)	0.001 (0.001)	−0.005 (0.002)
22	−0.002 (0.000)	−0.001 (0.000)	−0.002 (0.001)	−0.002 (0.000)	−0.003 (0.001)	−0.001 (0.000)	−0.007 (0.000)	−0.002 (0.000)
23	−0.020 (0.000)	−0.015 (0.003)	0.004 (0.003)	0.002 (0.004)	0.000† (0.000)	−0.004 (0.004)	−0.000 (0.003)	0.003 (0.004)
24	−0.003 (0.001)	−0.002 (0.001)	0.003 (0.000)	−0.003 (0.000)	−0.005 (0.001)	−0.006 (0.001)	−0.002 (0.000)	−0.007 (0.001)
25	0.002 (0.000)	0.002 (0.001)	0.009 (0.000)	0.001 (0.001)	0.000† (0.000)	−0.002 (0.001)	0.013 (0.000)	0.001 (0.002)
26	−0.006 (0.003)	−0.007 (0.002)	0.001 (0.001)	−0.006 (0.002)	−0.002 (0.003)	−0.005 (0.002)	−0.003 (0.001)	−0.008 (0.002)
27	−0.001 (0.002)	−0.007 (0.001)	−0.003 (0.001)	−0.006 (0.001)	−0.005 (0.002)	−0.004 (0.001)	−0.006 (0.001)	−0.004 (0.002)
28	0.027 (0.005)	0.017 (0.003)	0.002 (0.001)	0.026 (0.001)	0.041 (0.003)	0.032 (0.002)	0.007 (0.000)	0.068 (0.001)
29	−0.004 (0.000)	−0.003 (0.000)	0.006 (0.000)	−0.004 (0.000)	−0.002 (0.000)	−0.002 (0.000)	0.001 (0.000)	−0.003 (0.000)
30	0.022 (0.003)	−0.016 (0.003)	−0.001 (0.004)	−0.007 (0.005)	0.007 (0.002)	−0.007 (0.002)	−0.008 (0.003)	−0.007 (0.004)
31	−0.007 (0.002)	−0.031 (0.004)	−0.035 (0.004)	−0.033 (0.006)	−0.000† (0.000)	−0.054 (0.005)	−0.049 (0.005)	−0.060 (0.006)
32	−0.010 (0.001)	−0.010 (0.001)	−0.010 (0.001)	−0.008 (0.001)	−0.009 (0.001)	−0.008 (0.001)	−0.008 (0.001)	−0.005 (0.001)
33	−0.010 (0.003)	−0.013 (0.003)	−0.008 (0.002)	−0.009 (0.001)	−0.015 (0.002)	−0.019 (0.002)	0.038 (0.000)	−0.020 (0.004)
34	−0.017 (0.002)	−0.003 (0.000)	−0.002 (0.000)	−0.003 (0.002)	−0.025 (0.002)	−0.036 (0.003)	−0.019 (0.000)	−0.032 (0.002)
35	−0.010 (0.002)	−0.013 (0.001)	−0.000† (0.000)	−0.012 (0.001)	−0.007 (0.002)	−0.013 (0.001)	−0.013 (0.001)	−0.014 (0.001)
36	0.012 (0.000)	−0.007 (0.004)	0.003 (0.002)	−0.002 (0.003)	0.015 (0.004)	−0.005 (0.005)	0.008 (0.002)	0.001 (0.002)
37	−0.013 (0.001)	−0.015 (0.001)	−0.014 (0.001)	−0.013 (0.001)	−0.007 (0.001)	−0.009 (0.000)	−0.009 (0.001)	−0.008 (0.001)
38	−0.017 (0.007)	−0.110 (0.011)	−0.001 (0.003)	−0.104 (0.009)	−0.053 (0.006)	−0.045 (0.002)	−0.032 (0.000)	−0.059 (0.002)
39	−0.000† (0.000)	−0.023 (0.008)	−0.064 (0.027)	−0.034 (0.009)	0.003 (0.000)	−0.028 (0.002)	−0.026 (0.046)	−0.037 (0.006)
40	−0.014 (0.002)	−0.037 (0.002)	0.003 (0.000)	−0.010 (0.000)	−0.004 (0.003)	−0.029 (0.006)	0.005 (0.000)	−0.010 (0.000)
41	0.002 (0.000)	−0.032 (0.004)	−0.062 (0.038)	0.055 (0.000)	0.001 (0.000)	−0.031 (0.011)	−0.106 (0.024)	0.160† (0.000)
42	0.282† (0.000)	0.048 (0.015)	0.049 (0.012)	0.054 (0.012)	0.029 (0.000)	0.013 (0.023)	0.059 (0.007)	0.038 (0.031)

43	0.054 (0.002)	−0.045 (0.017)	0.013 (0.000)	0.003 (0.006)	−0.015 (0.003)	−0.001 (0.000)	−0.001 (0.000)	−0.030 (0.002)
44	−0.005 (0.004)	−0.011 (0.002)	−0.021 (0.003)	−0.015 (0.002)	−0.013 (0.002)	−0.016 (0.001)	−0.017 (0.001)	−0.014 (0.002)
45	−0.006 (0.001)	−0.007 (0.001)	−0.007 (0.001)	−0.005 (0.000)	−0.008 (0.001)	−0.009 (0.000)	−0.006 (0.001)	−0.007 (0.001)
46	−0.012 (0.002)	−0.012 (0.001)	−0.009 (0.001)	−0.004 (0.000)	−0.008 (0.002)	−0.015 (0.002)	−0.004 (0.000)	−0.004 (0.000)
47	−0.002 (0.000)	−0.003 (0.000)	0.003 (0.000)	0.001 (0.000)	−0.000 (0.001)	0.001 (0.000)	0.000† (0.000)	−0.006 (0.000)
48	−0.000† (0.000)	−0.064 (0.012)	−0.102 (0.023)	0.000† (0.000)	−0.021 (0.000)	−0.042 (0.014)	−0.164 (0.027)	−0.013 (0.000)
49	−0.006 (0.001)	−0.000† (0.000)	−0.007 (0.001)	−0.006 (0.000)	−0.006 (0.001)	−0.008 (0.000)	−0.007 (0.001)	0.000† (0.000)
50	−0.020 (0.002)	−0.025 (0.001)	0.006 (0.000)	−0.036 (0.000)	−0.010 (0.005)	−0.004 (0.000)	−0.004 (0.001)	−0.033 (0.002)

Notes: Bias computed as posterior mean minus true value. True values are different for each MC run. Standard deviation of posterior draws in parenthesis. A † indicates that something went wrong in the estimation, particularly a badly shaped inverse Hessian at the posterior mode which was used for the initial proposal covariance matrix.

Table 4: Bias for θ

	DGP UNPRUNED				DGP PRUNED			
MC run	CDKF	CDKFP	QKF	QKFP	CDKF	CDKFP	QKF	QKFP
Average	0.141 (0.292)	−0.274 (0.264)	0.140 (0.220)	0.025 (0.279)	0.092 (0.268)	−0.227 (0.240)	0.053 (0.233)	0.101 (0.274)
1	−0.290 (0.426)	−0.958 (0.504)	−1.149 (0.477)	−0.467 (0.452)	0.047† (0.000)	−0.545 (0.511)	−0.676 (0.443)	−0.562 (0.451)
2	0.114 (0.379)	−1.147 (0.266)	0.000† (0.000)	0.252 (0.371)	0.134 (0.383)	−0.256 (0.007)	0.101† (0.000)	−0.074 (0.030)
3	0.684 (0.399)	0.188 (0.023)	0.837 (0.497)	−0.132 (0.438)	0.589 (0.398)	−1.134 (0.134)	0.000† (0.000)	0.661 (0.387)
4	0.395 (0.425)	0.358 (0.398)	2.547 (0.777)	0.329 (0.406)	0.340 (0.407)	0.273 (0.363)	0.597 (0.370)	0.390 (0.386)
5	−0.619 (0.347)	−0.591 (0.057)	0.638† (0.000)	−1.145 (0.018)	−1.002 (0.315)	0.001† (0.000)	−0.768 (0.336)	2.651 (0.169)
6	−0.385 (0.488)	0.225 (0.433)	0.655 (0.289)	0.434 (0.434)	0.027 (0.301)	0.322 (0.443)	0.384 (0.289)	0.428 (0.433)
7	−0.897 (0.286)	2.420 (0.225)	0.000† (0.000)	−0.662 (0.294)	0.297 (0.260)	0.006† (0.000)	0.000† (0.000)	0.540 (0.148)
8	0.647 (0.161)	0.137 (0.336)	−1.434 (0.081)	−0.062† (0.000)	1.168 (0.232)	0.406 (0.031)	−0.298 (0.003)	0.599 (0.068)
9	0.386 (0.466)	−0.294 (0.093)	−0.575† (0.000)	0.113 (0.062)	0.142 (0.463)	0.395 (0.333)	−0.743 (0.243)	−0.068 (0.474)
10	−0.525 (0.061)	0.451 (0.408)	0.597 (0.466)	0.508 (0.386)	0.063 (0.271)	−0.141 (0.306)	−0.720 (0.297)	−0.412 (0.321)
11	0.039 (0.362)	−1.174 (0.016)	0.337† (0.000)	0.005 (0.059)	0.671 (0.285)	1.122 (0.168)	0.563† (0.000)	−0.062 (0.029)
12	−0.733 (0.440)	−0.516 (0.468)	−1.235 (0.317)	−0.807 (0.432)	−0.603 (0.452)	−0.696 (0.442)	−0.451 (0.456)	−0.704 (0.432)
13	0.529 (0.246)	−0.179 (0.074)	0.981 (0.335)	0.199 (0.385)	0.195 (0.064)	−0.372 (0.248)	0.861 (0.307)	−0.343 (0.348)
14	−0.394 (0.302)	−0.736 (0.316)	0.640 (0.307)	−0.315 (0.378)	−0.428 (0.415)	−0.418 (0.423)	0.480 (0.362)	−0.475 (0.408)
15	1.238 (0.473)	0.816 (0.445)	−0.358 (0.009)	1.231 (0.464)	0.297† (0.000)	−0.044 (0.009)	−1.001 (0.165)	0.791 (0.485)
16	−0.404 (0.382)	−1.690 (0.329)	−1.199† (0.000)	0.335 (0.442)	−0.835 (0.431)	−1.230 (0.366)	0.155† (0.000)	−0.320 (0.187)
17	0.545 (0.422)	0.670 (0.356)	1.970 (0.325)	0.791 (0.376)	0.137 (0.250)	0.146 (0.179)	0.957 (0.208)	−0.401 (0.266)
18	−0.293 (0.300)	−1.107 (0.007)	−0.541 (0.002)	1.107 (0.343)	−0.262 (0.336)	−0.381 (0.001)	−0.075† (0.000)	0.955 (0.299)
19	0.663 (0.204)	−0.107 (0.629)	0.624† (0.000)	−0.263 (0.446)	−0.285 (0.070)	−0.545 (0.079)	0.257 (0.450)	−0.363 (0.419)
20	0.468 (0.378)	−0.287 (0.275)	0.395 (0.367)	−0.061 (0.320)	0.415 (0.081)	0.054 (0.339)	0.447 (0.356)	0.128 (0.335)
21	0.501 (0.452)	0.306 (0.393)	1.481 (0.391)	0.409 (0.457)	0.533 (0.453)	0.193 (0.397)	0.845 (0.425)	0.501 (0.451)
22	0.230 (0.382)	0.160 (0.010)	−0.543 (0.073)	0.566 (0.034)	0.367 (0.390)	0.628 (0.082)	0.510† (0.000)	0.509 (0.050)
23	−0.615† (0.000)	−0.456 (0.482)	−0.662 (0.442)	−0.505 (0.435)	−0.084† (0.000)	−0.383 (0.479)	−0.370 (0.431)	−0.357 (0.461)
24	1.095 (0.429)	0.191 (0.332)	0.148† (0.000)	1.144 (0.179)	0.694 (0.419)	0.930 (0.408)	−0.414 (0.002)	1.284 (0.450)
25	0.016† (0.000)	−0.266 (0.456)	−0.821 (0.311)	−0.223 (0.479)	0.000† (0.000)	−0.473 (0.489)	2.160 (0.042)	−0.185 (0.483)

26	−0.279 (0.434)	−0.400 (0.378)	−0.570 (0.338)	−0.225 (0.420)	−0.392 (0.432)	−0.046 (0.473)	0.661 (0.380)	−0.443 (0.399)
27	0.884 (0.405)	0.544 (0.420)	2.220 (0.325)	0.783 (0.421)	0.769 (0.424)	0.093 (0.401)	2.527 (0.328)	0.655 (0.387)
28	0.067 (0.432)	0.629 (0.569)	−0.646 (0.094)	−0.254 (0.140)	1.023 (0.507)	0.457 (0.239)	0.039† (0.000)	−0.709 (0.313)
29	−0.119 (0.455)	−0.097 (0.033)	−0.368 (0.024)	−0.177 (0.256)	−0.030 (0.452)	−0.404 (0.246)	0.291 (0.007)	−0.054 (0.021)
30	0.029 (0.257)	0.675 (0.421)	1.781 (0.350)	0.901 (0.401)	0.004 (0.007)	1.169 (0.247)	1.359 (0.440)	0.990 (0.447)
31	−0.444 (0.181)	−0.178 (0.234)	0.727 (0.286)	−0.276 (0.292)	0.297† (0.000)	−0.696 (0.162)	0.505 (0.201)	−0.687 (0.220)
32	−0.405 (0.417)	−0.822 (0.431)	0.140 (0.335)	−0.321 (0.383)	−0.516 (0.430)	−0.940 (0.338)	0.067 (0.278)	−0.293 (0.382)
33	0.429 (0.229)	−0.045 (0.131)	0.802 (0.445)	0.520 (0.174)	−0.178 (0.109)	−0.415 (0.177)	−0.217 (0.162)	0.002 (0.061)
34	−0.059 (0.253)	−0.806 (0.001)	0.393† (0.000)	−0.267 (0.014)	−0.001 (0.279)	−1.146 (0.228)	−0.086† (0.000)	0.147 (0.088)
35	−0.133 (0.322)	−0.928 (0.188)	0.817† (0.000)	−0.435 (0.399)	0.229 (0.037)	−2.080 (0.154)	2.273 (0.233)	−1.558 (0.183)
36	0.648 (0.001)	−0.215 (0.516)	−0.433 (0.420)	−0.164 (0.447)	−0.064 (0.431)	−0.398 (0.502)	−0.445 (0.478)	−0.379 (0.446)
37	0.210 (0.292)	−1.013 (0.175)	0.500 (0.258)	0.055 (0.264)	0.797 (0.317)	−0.402 (0.139)	0.420 (0.188)	0.495 (0.221)
38	0.835 (0.244)	−0.751 (0.252)	−0.596 (0.109)	0.026 (0.024)	−0.533 (0.172)	0.092 (0.028)	−1.038† (0.000)	0.088 (0.194)
39	−0.471 (0.001)	−0.538 (0.219)	−1.394 (0.115)	0.706 (0.324)	−0.231 (0.001)	−0.718 (0.256)	−1.693 (0.302)	0.527 (0.396)
40	0.127 (0.020)	−0.701 (0.156)	−0.485† (0.000)	1.235 (0.070)	−0.048 (0.205)	−0.488 (0.088)	−0.044† (0.000)	0.622 (0.025)
41	0.890† (0.000)	−0.005 (0.140)	−1.597 (0.305)	−0.739 (0.001)	−0.000† (0.000)	0.347 (0.209)	−1.616 (0.484)	−0.273 (0.053)
42	−0.046 (0.009)	−1.396 (0.227)	−0.087 (0.437)	−0.552 (0.470)	0.309† (0.000)	−1.726 (0.328)	−0.519 (0.460)	−0.158 (0.564)
43	2.468 (0.317)	−1.404 (0.419)	0.178† (0.000)	−0.837 (0.411)	0.698 (0.364)	−0.237 (0.006)	−0.064† (0.000)	0.598 (0.303)
44	−0.267 (0.142)	−1.185 (0.194)	0.946 (0.286)	−1.233 (0.238)	−0.644 (0.419)	−0.759 (0.412)	0.039 (0.389)	−0.689 (0.443)
45	0.142 (0.385)	−0.580 (0.267)	−0.043 (0.187)	0.705 (0.333)	0.238 (0.408)	−0.719 (0.274)	0.120 (0.161)	0.576 (0.331)
46	0.082 (0.409)	−0.411 (0.038)	−1.454 (0.153)	0.040 (0.004)	0.278 (0.568)	−0.714 (0.234)	−0.441† (0.000)	0.130† (0.000)
47	−0.413 (0.480)	−0.061 (0.050)	0.837 (0.150)	−0.654 (0.100)	−0.284 (0.486)	0.358 (0.042)	0.000† (0.000)	−0.720 (0.097)
48	0.000† (0.000)	0.246 (0.233)	1.699 (0.775)	0.000† (0.000)	0.400† (0.000)	1.782 (0.344)	−0.256† (1.165)	0.186† (0.000)
49	−0.231 (0.287)	−0.432† (0.000)	0.324 (0.154)	−0.266 (0.189)	−0.348 (0.263)	−1.412 (0.231)	−0.657 (0.123)	0.616 (0.012)
50	0.707 (0.394)	−0.221 (0.178)	−0.035† (0.000)	−0.079 (0.062)	0.182 (0.433)	−0.197 (0.001)	−1.388 (0.662)	0.265 (0.161)

Notes: Bias computed as posterior mean minus true value. True values are different for each MC run. Standard deviation of posterior draws in parenthesis. A † indicates that something went wrong in the estimation, particularly a badly shaped inverse Hessian at the posterior mode which was used for the initial proposal covariance matrix.

Table 5: Bias for ϕ

	DGP UNPRUNED				DGP PRUNED			
MC run	CDKF	CDKFP	QKF	QKFP	CDKF	CDKFP	QKF	QKFP
Average	0.059 (0.152)	−0.269 (0.157)	0.419 (0.143)	−0.005 (0.174)	−0.020 (0.139)	−0.167 (0.158)	0.251 (0.134)	0.050 (0.173)
1	0.597 (0.361)	−2.215 (0.598)	−0.499 (0.866)	−0.282 (0.364)	−0.020† (0.000)	−0.896 (0.453)	−0.931 (0.219)	−0.278 (0.382)
2	−0.064 (0.128)	−0.457 (0.101)	−0.000† (0.000)	0.189 (0.169)	−0.049 (0.132)	−0.112 (0.005)	0.014† (0.000)	−0.110 (0.018)
3	0.213 (0.123)	0.083 (0.007)	0.832 (0.212)	−0.033 (0.148)	0.200 (0.126)	−0.385 (0.044)	0.000† (0.000)	0.259 (0.131)
4	0.713 (0.369)	0.478 (0.336)	0.080 (0.009)	0.447 (0.341)	0.332 (0.288)	0.283 (0.301)	0.780 (0.314)	0.632 (0.347)
5	−0.546 (0.206)	−0.397 (0.058)	−0.001† (0.000)	−0.706 (0.030)	−0.804 (0.177)	−0.000† (0.000)	−0.130 (0.060)	2.141 (0.146)
6	0.348 (0.231)	0.464 (0.254)	1.663 (0.223)	0.571 (0.266)	0.874 (0.218)	0.047 (0.236)	1.436 (0.229)	0.618 (0.251)
7	−1.053 (0.194)	1.433 (0.190)	0.000† (0.000)	−0.787 (0.199)	−0.147 (0.184)	−0.037† (0.000)	−0.001† (0.000)	0.065 (0.101)
8	0.217 (0.030)	−0.090 (0.087)	0.760 (0.265)	−0.057† (0.000)	0.379 (0.070)	0.101 (0.001)	0.265 (0.002)	0.112 (0.013)
9	0.024 (0.092)	−0.112 (0.019)	0.671† (0.000)	0.016 (0.011)	−0.053 (0.088)	0.074 (0.081)	1.238 (0.152)	−0.187 (0.068)
10	0.237 (0.039)	−0.304 (0.208)	−0.431 (0.195)	0.015 (0.413)	1.008 (0.300)	0.387 (0.302)	−0.100 (0.212)	0.101 (0.288)
11	−0.196 (0.228)	−0.882 (0.013)	1.101† (0.000)	−0.032 (0.032)	0.246 (0.171)	0.643 (0.121)	−0.058† (0.000)	−0.003† (0.000)
12	−0.306 (0.264)	−0.140 (0.333)	0.394 (0.271)	−0.150 (0.306)	−0.532 (0.249)	−0.051 (0.321)	−0.193 (0.296)	−0.053 (0.283)
13	0.311 (0.176)	−0.402 (0.081)	0.408 (0.258)	−0.053 (0.308)	0.044 (0.119)	−0.367 (0.206)	0.566 (0.273)	−0.282 (0.322)
14	0.309 (0.285)	0.127 (0.342)	0.266 (0.204)	−0.143 (0.315)	−0.505 (0.247)	−0.408 (0.289)	0.418 (0.253)	−0.280 (0.318)
15	0.267 (0.096)	0.203 (0.097)	0.884 (0.010)	0.134 (0.078)	0.049† (0.000)	−0.030 (0.010)	−0.157 (0.039)	0.038 (0.065)
16	−0.210 (0.194)	−0.893 (0.158)	0.008† (0.000)	−0.087 (0.299)	−0.617 (0.166)	−0.716 (0.154)	0.655† (0.000)	−0.383 (0.059)
17	0.375 (0.238)	0.347 (0.221)	1.130 (0.182)	0.412 (0.227)	0.231 (0.146)	0.346 (0.130)	1.549 (0.196)	−0.351 (0.104)
18	−0.231 (0.132)	−0.616 (0.010)	0.038† (0.000)	0.690 (0.189)	−0.190 (0.153)	−0.233 (0.002)	0.026† (0.000)	0.459 (0.172)
19	1.205 (0.279)	−0.179 (0.465)	−0.444† (0.000)	−0.011 (0.391)	−0.625 (0.018)	−0.245 (0.210)	−1.000 (0.213)	−0.011 (0.395)
20	−0.116 (0.318)	−0.280 (0.359)	0.138 (0.358)	−0.193 (0.338)	0.061 (0.061)	−0.035 (0.366)	−0.218 (0.289)	−0.003 (0.379)
21	0.252 (0.201)	0.065 (0.144)	0.601 (0.194)	0.156 (0.188)	0.210 (0.196)	−0.054 (0.131)	0.509 (0.230)	0.144 (0.210)
22	0.026 (0.231)	0.105 (0.030)	1.133 (0.351)	0.341 (0.025)	0.053 (0.224)	0.434 (0.036)	1.554† (0.000)	0.436 (0.050)
23	0.068 (0.001)	−0.247 (0.213)	−0.244 (0.160)	0.306 (0.322)	−0.041† (0.000)	−0.324 (0.452)	0.406 (0.320)	0.055 (0.304)
24	0.424 (0.191)	0.123 (0.171)	1.888† (0.000)	0.422 (0.082)	0.205 (0.183)	0.365 (0.194)	−0.041† (0.000)	0.492 (0.225)
25	0.012† (0.000)	−0.032 (0.036)	−0.074 (0.022)	−0.078 (0.038)	−0.000† (0.000)	−0.050 (0.058)	−0.215† (0.000)	−0.127 (0.049)

26	−0.162 (0.190)	−0.270 (0.174)	−0.119 (0.173)	−0.138 (0.212)	−0.154 (0.206)	0.027 (0.259)	0.353 (0.204)	−0.169 (0.203)
27	0.435 (0.206)	0.180 (0.196)	1.321 (0.201)	0.303 (0.203)	0.271 (0.203)	0.001 (0.205)	1.092 (0.193)	0.259 (0.213)
28	0.066 (0.056)	−0.031 (0.076)	0.136 (0.008)	0.129 (0.013)	0.007 (0.059)	0.057 (0.024)	−0.003† (0.000)	0.209 (0.046)
29	−0.095 (0.101)	−0.053 (0.011)	0.699 (0.040)	−0.034 (0.064)	−0.061 (0.102)	−0.107 (0.061)	0.815 (0.005)	0.075 (0.009)
30	0.255 (0.150)	−0.042 (0.101)	0.606 (0.165)	0.254 (0.181)	0.010 (0.025)	0.352 (0.131)	0.259 (0.162)	0.287 (0.224)
31	−0.399 (0.148)	−0.510 (0.188)	−0.171 (0.172)	−0.547 (0.239)	0.485 (0.001)	−0.929 (0.115)	−0.043 (0.146)	−0.963 (0.128)
32	−0.417 (0.222)	−0.533 (0.254)	1.098 (0.264)	−0.140 (0.246)	−0.466 (0.230)	−0.487 (0.209)	1.338 (0.279)	−0.155 (0.241)
33	0.095 (0.057)	−0.058 (0.034)	−0.126 (0.012)	0.084 (0.027)	−0.007 (0.032)	−0.134 (0.032)	0.063 (0.001)	−0.097 (0.022)
34	−0.079 (0.217)	−0.787 (0.003)	0.718† (0.000)	−0.224 (0.073)	−0.251 (0.213)	−1.188 (0.207)	0.050† (0.000)	−0.104 (0.047)
35	−0.249 (0.177)	−0.689 (0.104)	0.894† (0.000)	−0.374 (0.246)	0.058 (0.038)	−1.305 (0.089)	0.982 (0.144)	−1.027 (0.100)
36	0.249 (0.001)	−1.003 (0.404)	0.229 (0.336)	−0.232 (0.315)	0.009 (0.307)	−0.124 (0.296)	−0.085 (0.366)	0.030 (0.349)
37	−0.161 (0.213)	−1.011 (0.149)	1.922 (0.270)	−0.184 (0.218)	0.439 (0.257)	−0.444 (0.127)	2.262 (0.244)	0.301 (0.206)
38	0.232 (0.172)	−1.421 (0.248)	−0.313 (0.088)	−0.145 (0.010)	−0.984 (0.169)	−0.122 (0.118)	−1.302† (0.000)	0.331 (0.319)
39	−0.169† (0.000)	−0.259 (0.114)	0.757 (0.178)	−0.085 (0.120)	0.072 (0.001)	−0.390 (0.081)	0.064 (0.218)	−0.162 (0.102)
40	0.257 (0.019)	−0.411 (0.065)	−0.356† (0.000)	0.511 (0.021)	0.055 (0.106)	−0.330 (0.048)	0.081† (0.000)	−0.002 (0.009)
41	0.250† (0.000)	−0.246 (0.060)	−0.392 (0.147)	0.067† (0.000)	0.000† (0.000)	−0.012 (0.143)	−0.899 (0.278)	0.506 (0.038)
42	0.050 (0.005)	0.401 (0.481)	−0.515 (0.175)	0.557 (0.388)	0.021† (0.000)	0.114 (0.466)	−0.798 (0.148)	−0.751† (1.002)
43	0.932 (0.107)	−0.383 (0.120)	0.021† (0.000)	0.112 (0.410)	0.138 (0.092)	0.344 (0.001)	0.085† (0.000)	−0.157 (0.039)
44	−0.290 (0.050)	−0.893 (0.144)	0.183 (0.199)	−0.995 (0.164)	−0.680 (0.249)	−0.697 (0.267)	−0.510 (0.216)	−0.492 (0.338)
45	−0.085 (0.203)	−0.394 (0.161)	1.860 (0.274)	0.417 (0.206)	−0.004 (0.224)	−0.488 (0.167)	2.358 (0.252)	0.243 (0.200)
46	−0.238 (0.182)	−0.380 (0.025)	1.082 (0.437)	0.020 (0.003)	0.149 (0.324)	−0.451 (0.134)	−0.140† (0.000)	0.067† (0.000)
47	−0.038 (0.025)	−0.015 (0.006)	0.099 (0.008)	−0.033 (0.007)	−0.016 (0.029)	0.031 (0.004)	−0.000† (0.000)	−0.128 (0.002)
48	−0.000† (0.000)	−0.217 (0.070)	−0.666 (0.054)	0.000† (0.000)	0.132† (0.000)	1.129 (0.339)	−0.496 (0.194)	0.023† (0.000)
49	−0.534 (0.302)	−0.375† (0.000)	1.837 (0.163)	−0.361 (0.249)	−0.643 (0.279)	−1.754 (0.285)	1.537 (0.273)	0.538 (0.018)
50	0.152 (0.181)	−0.225 (0.119)	−0.161† (0.000)	−0.318 (0.026)	0.114 (0.266)	−0.182 (0.001)	−0.894 (0.098)	0.369 (0.138)

Notes: Bias computed as posterior mean minus true value. True values are different for each MC run. Standard deviation of posterior draws in parenthesis. A † indicates that something went wrong in the estimation, particularly a badly shaped inverse Hessian at the posterior mode which was used for the initial proposal covariance matrix.