Baseline New-Keynesian Model

Optimal Monetary Policy

(Short version)

Motivation

Generally accepted principle: policymakers should act in an optimal manner, i.e. maximize welfare for representative household

DSGE models enable one to

- compare different policy rules
- compute optimal policies (given a pre-defined objective)
- quantify welfare gain/loss of different policies and instruments

Canonical New-Keynesian Model

Two sources of distortions/inefficiencies

- Market power due to monopolistically competitive firms
- Relative price distortions resulting from staggered price setting

Inefficiencies are analytically tractable and quantifiable

Optimal allocation is equal to undistorted flex-price allocation

Definitions

Efficient output y_t^e: Level of output that would prevail under perfect competition

Natural output y_t^n: Level of output that would prevail under imperfect competition, but flexible prices and wages

Characterize Optimal Policy

Optimal policy requires:

- subsidy financed by lump-sum taxes that offsets market power
  $\hookrightarrow$ optimal employment subsidy yields efficient allocation

- monetary policy rule that stabilizes marginal costs at a level consistent with firms' desired markup at unchanged prices
  $\hookrightarrow$ aggregate price stability yields natural flex-price allocation

- In sum: $y_t^e = y_t^n$

What about output? Is stabilizing output ($var(y_t) = 0$) desirable?

Equilibrium Under Optimal Policy

Consider non-policy block:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$
$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right)$$

Optimal allocation: $\tilde{y}_t = y_t - y_t^n = 0$ and $\pi_t = 0$ and $i_t = r_t^n$

Achieving $\pi_t = 0$ implies $\tilde{y}_t = 0$ and $i_t = r_t^n$

No policy trade-off between output and inflation stabilization
↪ "Divine Coincidence"

Implementing Optimal Policy

Why not exogenous one-for-one rule $i_t = r_t^n$?

Equilibrium dynamics: $$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1/\sigma \\ \kappa & \beta + \kappa/\sigma \end{bmatrix}}_{A_0} E_t \begin{bmatrix} \widetilde{y}_{t+1} \\ \pi_{t+1} \end{bmatrix}$$

One eigenvalue of A_0 is strictly greater than one

$\hookrightarrow$ indeterminacy (nominal and real)

BUT: Optimal allocation features uniqueness and determinacy

Implementing Optimal Policy

Interest rate rule with feedback to target variables: $i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \underbrace{(\sigma + \phi_y + \kappa\phi_\pi)^{-1} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}}_{A_T} E_t \begin{bmatrix} \widetilde{y}_{t+1} \\ \pi_{t+1} \end{bmatrix}$$

Unique and determinate solution: $\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$

Taylor principle ($\phi_\pi > 1$) always ensures stable and unique solution

Policy Trade-Offs

In reality policy makers face trade-offs (at least in the short-run) due to several sources of uncertainty and frictions

Usually central bankers commit to medium-term inflation target, but also want to avoid excessive instability of output and employment

But: do they commit to their plans?

- Commitment: make state-contingent policy plan, bound by past promises
- Discretion: make decision each period, don't feel bound by past promises

Farewell Divine Coincidence

Definitions:

- Efficient output y_t^e: Level of output that would prevail under perfect competition

- Natural output y_t^n: Level of output that would prevail under imperfect competition, but flexible prices and wages

Nominal rigidities AND real frictions break divine coincidence as flex-price allocation is inefficient and not optimal to target

Farewell Divine Coincidence

Assume: efficient steady-state, but time-varying gap between efficient and natural output

$$u_t = \kappa(y_t^e - y_t^n) = \rho_u u_{t-1} + \varepsilon_{u,t}$$

Non-policy block (with welfare-relevant output gap $x_t = y_t - y_t^e$):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

$$x_t = E_t x_{t+1} - 1/\sigma(i_t - E_t \pi_{t+1} - r_t^e)$$

Welfare losses by representative household, up to a second-order approximation, proportional to

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right) \text{ where } \vartheta = \frac{\kappa}{\epsilon}$$

Ramsey Problem

Social planner maximizes objective function or minimizes loss function by choosing specific policy instrument and taking into account the equilibrium conditions of the economy

FOC of social planner's problem and equilibrium conditions form a system of equations with

- endogenous variables
- policy instrument(s)
- Lagrangian multipliers of the social planner's problem
- initial conditions

Linearized system then enables one to compute e.g. impulse responses to a given shock

Ramsey Problem

Objective depends on how well policy maker "keeps promises":

Commitment vs Discretion

Downside of Ramsey approach:

- Which policy instrument?
- Inefficiencies are generally not tractable
- In general communicating optimal policy not straightforward

Ramsey Problem

Major difficulty:

- Computing a steady state solution CONDITIONAL on the value of the instruments in the optimal policy problem
- initial values of the instruments
- parameters also need to be updated during steady state computations

Optimal Policy Under Commitment

At time 0, policy maker chooses a state-contingent policy $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2)$$

subject to sequence of constraints $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$

Algebra: set up Lagrangian and get FOC

Optimal Policy Under Commitment
Dynare Commands

`planner_objective`:

- declare (one-period, not discounted lifetime) objective of policy maker
- can be arbitrary nonlinear expressions; not limited to quadratic objectives which focus on (co-)variances only

`ramsey_model`

- creates expanded model, i.e. computes FOC (and Lagrange multipliers) for maximizing objective subject to constraints provided by equilibrium conditions
- further computations are needed to be run (e.g. steady, deterministic or stochastic simulations, estimation)
- `planner_discount`: discount factor of objective function
- `instruments`: instrument variables for computation of steady state under optimal policy (requires analytical steady state block or file)

`ramsey_constraints`: constraints on the variables, e.g. (`i>0`)

`evaluate_planner_objective`: computes, displays, and stores the value of the planner objective function under Ramsey policy

`ramsey_policy`: equivalent to running "`ramsey_model; stoch_simul(order=1); evaluate_planner_objective;`"

Optimal Policy Under Discretion

Operates sequentially, i.e. each period choosing (x_t, π_t) to minimize

$$\pi_t^2 + \vartheta x_t^2$$

subject to $\pi_t = \kappa x_t + \nu_t$, where $\nu_t = \beta E_t \pi_{t+1} + u_t$ can be taken as given

$$\text{Optimality: } x_t = -\frac{\kappa}{\vartheta}\pi_t$$

Optimal Policy Under Discretion
Dynare Commands

`planner_objective`:

- declare (one-period, not discounted lifetime) objective of policy maker
- limited to quadratic objectives, i.e. focusing on (co-)variances only
- ensure linear model by setting `model(linear);`

`discretionary_policy`:

- computes an approximation to optimal policy under discretion
- essentially a LQ solver

Remarks

Optimal Policy under Commitment

- By promising future output gaps, the CB can improve the inflation/ output-gap trade-off today
- Given convexity of loss function, this improves welfare

Optimal Policy under Discretion

- "Stabilization Bias", i.e. CB stabilizes output gap in the medium term "more than it should" (compared to commitment)

Remarks

How do we communicate the simple rules or the Ramsey policy to central bankers?

Need to know natural rate of interest or efficient rate of interest, requires knowledge of true model, true parameter values, realized shocks (that affect r_t^n and r_t^e)

$\hookrightarrow$ Simple rules and Ramsey policy often infeasible to communicate or recommend

Simple Implementable Rules

Alternative: *simple implementable rules*

- Policy instrument depends on observable variables only
- Do not require knowledge of true parameter values
- Need to come up with an evaluation criteria

Comparing Policy Regimes

Define objective that depends on policy, e.g.

- Conditional welfare: $W_t = E_t \sum_{j=0}^{\infty} \beta^j U(c_t, n_t) = U(c_t, n_t) + \beta E_t W_{t+1}$

- Unconditional welfare mean: $\mathscr{W}_0 = E \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$

- Loss function: $E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2)$

Comparing Policy Regimes

Consumption Equivalent

- Quantify welfare differences between optimized rules and a baseline rule by computing the compensating fraction Ω of steady-state consumption

- Idea: how much steady-state consumption would be necessary to equate the level of welfare in a baseline scenario to the level of welfare under an alternative scenario

$$E_t \sum_{j=1}^{\infty} \beta^j U((1+\Omega)c^b_{t+j}, n^b_{t+j}) = W^o_t$$

- For additive-separable CES utility, after some algebra:

$$\Omega = [(W^o_t - W^b_t)(1-\sigma)(1-\beta)c^{\sigma-1} + 1]^{\frac{1}{1-\sigma}} - 1$$

Comparing Policy Regimes

How to compare different policy regimes

- start at same initial condition (e.g. steady-state)
- approximate model to at least 2nd order
- define grid for parameters (e.g. parameters of policy rule)
- search parameters by optimizing objective

Optimal Simple Implementable Rules

Policy instrument is a linear function of a few observable variables of the model

Parameters of optimal policy rule can be easily communicated to policy makers

Numerically optimize parameters of simple implementable rules (implies commitment)

Downside:

- What is the policy instrument?
- What are the variables that show up in the policy rule?

Optimal Simple Implementable Rules
Dynare Commands

`osr` : computes optimal simple policy rules for linear-quadratic problem:

$$\min_{\gamma} E(y_t' W y_t)$$

$$\text{such that } g_+ E_t y_{t+1} + g_0 y_t + g_- y_{t-1} + g_u u_t = 0$$

`osr_params`: list of parameters γ

`osr_params_bounds`: optional bounds for γ

`optim_weights`: selecting subset of variables by attaching weights to only these