Baseline New-Keynesian Model

Zero-Lower-Bound (Short version)

Motivation

Occasionally binding constraints are all over the place in Macro

Most prominent: Zero-Lower-Bound on nominal interest rates

What does this imply for welfare and optimal monetary policy?

Framework: Baseline New Keynesian model with divine coincidence

Divine Coincidence

New Keynesian Model with Divine Coincidence

Abstract from real imperfections and consider canonical representation:

New-Keynesian Phillips curve: $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$

New-Keynesian dynamic IS curve: $x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right)$

Interest rate rule: $i_t = \max\{0, r_t^n + \phi_\pi \pi_t\}$

Optimal allocation: $x_t = y_t - y_t^n = 0$ and $\pi_t = 0$ and $i_t = r_t^n$

No policy trade-off between output and inflation stabilization

↪ "Divine Coincidence": Achieving $\pi_t = 0$ implies $x_t = 0$ and $i_t = r_t^n$

↪ Taylor principle ($\phi_\pi > 1$) always ensures stable and unique solution

Zero-Lower-Bound

Zero-Lower-Bound

Natural rate of interest: $r_t^n = -\sigma(1-\rho_a)\psi_{ya}a_t + (1-\rho_z)z_t$

Decline of r_t^n (negative natural rate shock) due to a

- positive productivity shock on a_t
- negative demand shock on z_t

Drop in r_t^n may induce zero-lower-bound as $i_t = \max\{0, r_t^n + \phi_\pi \pi_t\})$

Zero-Lower-Bound

Assume following path for $r_t^n = \begin{cases} 0 & \text{for } t < 0 \\ -\epsilon < 0 & \text{for } 0 \leq t \leq t_Z \\ 0 & \text{for } t > t_Z \end{cases}$

What is the optimal response under discretion vs commitment that minimizes the loss function

$$\frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right)$$

Optimal Monetary Policy

Discretion

$$\mathscr{L} = \frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_{1,t}\left(\pi_t - \kappa x_t - \beta E_t \pi_{t+1}\right) + \xi_{2,t}\left(x_t - E_t x_{t+1} + \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n\right)$$

Combine $\dfrac{\partial \mathscr{L}}{\partial \pi_t} = 0$ and $\dfrac{\partial \mathscr{L}}{\partial x_t} = 0$ to get: $\vartheta x_t = -\kappa \pi_t - \xi_{2,t}$

Mixed complementary problem

- $\dfrac{\partial \mathscr{L}}{\partial i_t} = \xi_{2,t} \dfrac{1}{\sigma} = 0$ must hold whenever $i_t > 0$
- alternatively: complementary slackness condition $\xi_{2,t} i_t = 0$

Commitment

$$\mathscr{L} = E_0 \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{2} \left(\pi_s^2 + \vartheta x_s^2 \right) + \xi_{1,s} \left(\pi_s - \kappa x_s - \beta \pi_{s+1} \right) + \xi_{2,s} \left(x_s - x_{s+1} + \frac{1}{\sigma} \left(i_s - \pi_{s+1} - r_s^n \right) \right) \right]$$

First-order condition wrt π_t: $\dfrac{\partial \mathscr{L}}{\partial \pi_t} = \pi_t + \xi_{1,t} - \xi_{1,t-1} - \dfrac{1}{\beta\sigma}\xi_{2,t-1} = 0$

First-order condition wrt x_t: $\dfrac{\partial \mathscr{L}}{\partial x_t} = \vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \dfrac{1}{\beta}\xi_{2,t-1} = 0$

Mixed complementary problem:

- $\dfrac{\partial \mathscr{L}}{\partial i_t} = \xi_{2,t}\dfrac{1}{\sigma} = 0$ must hold whenever $i_t > 0$
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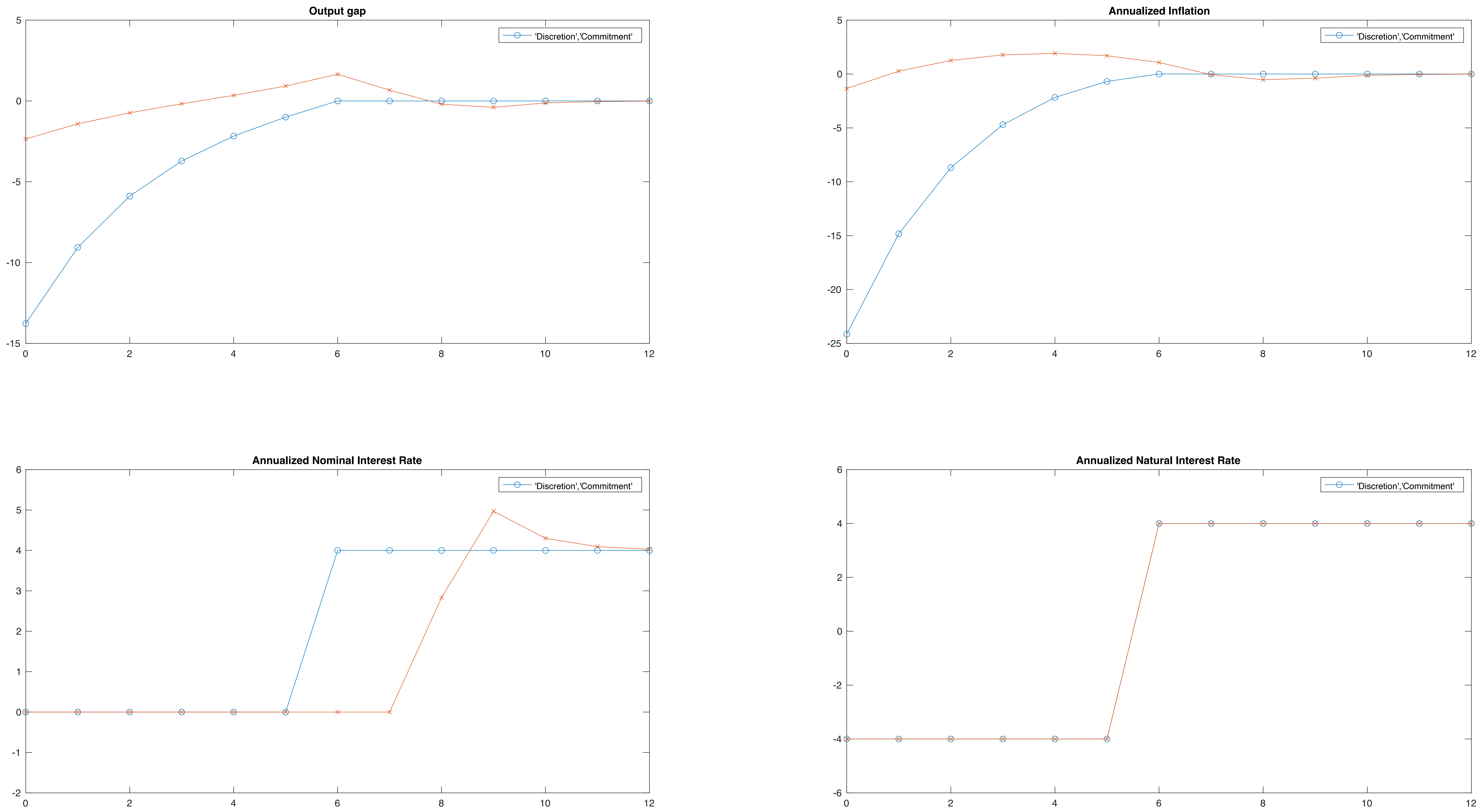
Output gap
'Discretion','Commitment'
Annualized Inflation
'Discretion','Commitment'
Annualized Nominal Interest Rate
'Discretion','Commitment'
Annualized Natural Interest Rate
'Discretion','Commitment'

Dynare Implementation

Occasionally binding constraints in Dynare

Models with OBC may be simulated

- under perfect foresight using `max/min` operators or (better) `lmmcp`
- in a stochastic framework using toolboxes like occbin

Difficulties

- existence and uniqueness of solution
- algorithms with reliable accuracy AND sufficient speed

Occasionally binding constraints in Dynare

`max`/`min` operators can be used with deterministic simulations, but yield singular Jacobians (in stochastic frameworks `max`/`min` are ignored)

Levenberg-Marquardt mixed complementarity problem

- slackness condition described by equation tag `mcp`
- MCP solver triggered with `perfect_foresight_solver(lmmcp)`

occbin for linearized stochastic models is coming in Dynare 4.7

Summary

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Zero-lower-bound may constrain monetary policy

Adverse dynamics if monetary policy lacks ability to commit

Natural rate low/negative for a variety of reasons