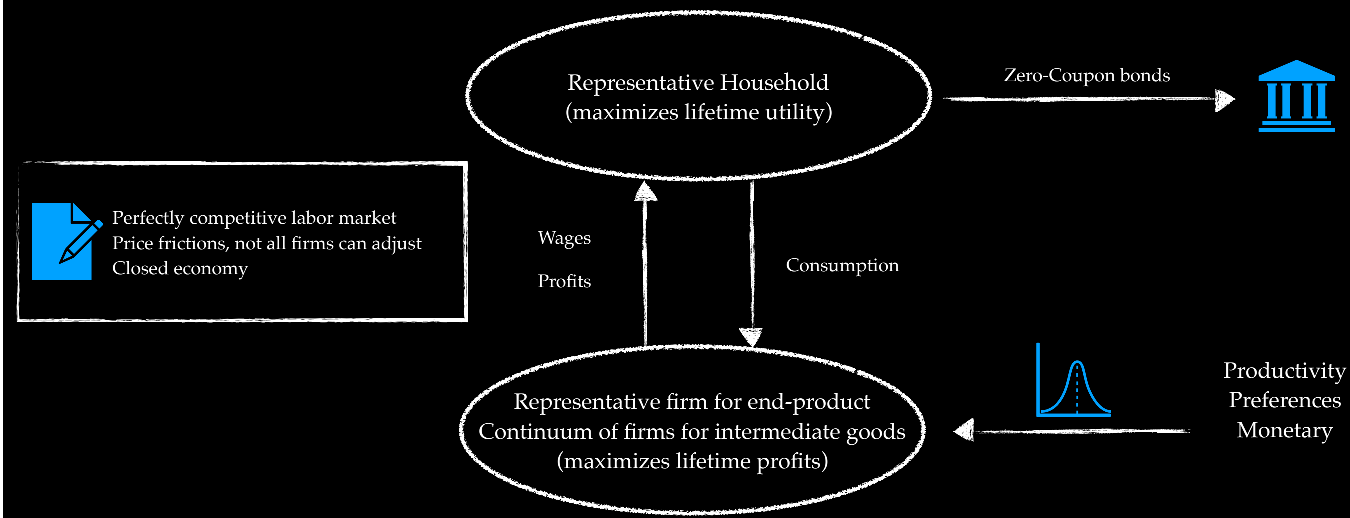
Basic New Keynesian Model

Model description

Model structure



Household

Representative household maximizes present as well as expected future utility

$$\max E_t \sum_{j=0}^{\infty} \beta^j U(c_{t+j}, n^s_{t+j}, z_{t+j})$$

Functional forms

$$U(c_t, n^s_t, z_t) = \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{(n^s_t)^{1+\varphi}}{1+\varphi} \right) z_t$$

Consumption index

Dixit-Stiglitz (1977) aggregation technology:

$$c_t = \left(\int_0^1 c_t(h)^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \qquad \epsilon > 1$$

Household

Budget constraint (in nominal terms)

$$\int_0^1 P_t(h)c_t(h)dh + Q_tB_t \leq B_{t-1} + W_tn_t^s + P_t\int_0^1 div_t(f)df$$

Interest rates

Nominal interest rate:

$$Q_t = \frac{1}{R_t}$$

Real interest rate:

$$R_t = r_tE_t\Pi_{t+1}$$

Debt

Stochastic discount factor:

$$\Lambda_{t,T} = \beta^{T-t} \frac{\partial U(c_T, n_T^s, z_T)/\partial c_T}{\partial U(c_t, n_t^s, z_T)/\partial c_t}$$

Solvency constraint (no Ponzi-type schemes)

$$\lim_{T\to\infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} \geq 0$$

Debt

Solvency constraint (no Ponzi-type schemes)

$$\lim_{T\to\infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} \geq 0$$

Transversality condition

$$\lim_{T\to\infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} = 0$$

Consumption cost minimization

$$c_t(h) = \left(\frac{P_t(h)}{P_t} \right)^{-\epsilon} c_t \qquad \text{and} \qquad P_t = \left(\int_0^1 P_t(h)^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}}$$

Implication for budget constraint: $\int_0^1 P_t(h) c_t(h) dh = P_t c_t$

Household optimality

$$w_t := \frac{W_t}{P_t} = - \frac{\frac{\partial U(c_t, n_t^s, z_t)}{\partial n_t^s}}{\frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t}}$$

$$\frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t} = \beta E_t \left[\frac{\partial U(c_{t+1}, n_{t+1}^s, z_{t+1})}{\partial c_{t+1}} r_t \right]$$

Firms

Stochastic discount factor

Firms are owned by households

Stochastic discount factor to evaluate profits:

$$E_t \Lambda_{t,t+j} = E_t 1/R_{t+j} = E_t \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{P_t}{P_{t+j}}$$

Stochastic discount factor

Some special cases:

$$\Lambda_{t,t} = 1$$

$$\Lambda_{t+1,t+1+j} = \beta^j \frac{\lambda_{t+1+j}}{\lambda_{t+1}} \frac{P_{t+1}}{P_{t+1+j}}$$

$$\Lambda_{t,t+1+j} = \beta^{j+1} \frac{\lambda_{t+1+j}}{\lambda_t} \frac{P_t}{P_{t+1+j}} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \beta^j \frac{\lambda_{t+1+j}}{\lambda_{t+1}} \frac{P_{t+1}}{P_{t+1+j}} = \beta \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{-1} \Lambda_{t+1,t+1+j}$$

Firm: final product aggregation

Dixit-Stiglitz (1977) aggregation technology:

$$y_t = \left[\int_0^1 y_t(f)^{\frac{\epsilon - 1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon - 1}}$$

Profit maximization:

$$y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon} y_t \qquad \text{and} \qquad P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}$$

Firms: intermediate goods

linear production function: $y_t(f) = a_t n_t^d(f)$

real profits: $div_t(f) = \frac{P_t(f)}{P_t} y_t(f) - w_t n_t^d(f)$

present value of nominal dividends: $E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} P_{t+j} div_{t+j}(f)$

Firms: intermediate goods

optimal labor demand:

$$w_t = mc_t(f) a_t = mc_t(f) \frac{y_t(f)}{n_t^d(f)}$$

Implications for real marginal costs:

$$mc_t = \int_0^1 mc_t(f) df = \frac{w_t}{a_t}$$

Firms: intermediate goods

Calvo (1983) and Yun (1996) nominal rigidities:

In each period firm f faces a constant probability $1-\theta$ of being able to re-optimize its prize $P_t(f)$:

$$P_t(f) = \begin{cases} \widetilde{P}_t(f) & \text{with probability } 1-\theta \\ P_{t-1}(f) & \text{with probability } \theta \end{cases}$$

where $\widetilde{P}_t(f)$ is the re-optimized price in period t

Firms: intermediate goods

Probability to be stuck at same price for j periods is θ^j

Objective: maximize expected profits until firm can re-optimize the price again in some future period t+j

Firms: intermediate goods

optimal price setting:

$$\widetilde{p}_t \cdot s_{1,t} = \frac{\epsilon}{\epsilon - 1} \cdot s_{2,t} \quad \text{where } \widetilde{p}_t := \frac{\widetilde{P}_t(f)}{P_t}$$

$$s_{1,t} = y_t \frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t} + \beta\theta E_t \Pi_{t+1}^{\epsilon-1} s_{1,t+1}$$

$$s_{2,t} = mc_t y_t \frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t} + \beta\theta E_t \Pi_{t+1}^{\epsilon} s_{2,t+1}$$

Firms: intermediate goods

law of motion for optimal reset price $\widetilde{p}_t := \widetilde{P}_t(f)/P_t$:

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta)\,\widetilde{p}_t^{1-\epsilon}$$

Market clearing

Market clearing

Bond market: $B_t = 0$

Labor market: $n_t^s = n_t = \int_0^1 n_t^d(f)df$

Aggregate real profits: $div_t \equiv \int_0^1 div_t(f)df = y_t - w_t n_t$

Aggregate demand: $y_t = c_t$

Market clearing

Aggregate supply:

$$p_t^* y_t = a_t n_t$$

Price inefficiency distortion:

$$p_t^* = \int_0^1 \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon} df$$
$$p_t^* = (1-\theta)\,\widetilde{p}_t^{-\epsilon} + \theta \Pi_t^{\epsilon} p_{t-1}^*$$

Monetary policy

Monetary Policy

Taylor rule:

$$R_t = R \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{y_t}{y} \right)^{\phi_y} e^{\nu_t}$$

Stochastic processes

Exogenous variables

Preference shifter:

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t}$$

Productivity:

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}$$

Monetary policy shock:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t}$$

Exogenous variables

Stochastic shocks are Gaussian:

$$\begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{a,t} \\ \varepsilon_{\nu,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_a^2 & 0 \\ 0 & 0 & \sigma_\nu^2 \end{pmatrix} \right)$$

Nonlinear model equations

Nonlinear model equations

$$Q_t = \frac{1}{R_t} \qquad mc_t = \frac{w_t}{a_t} \qquad y_t = c_t \qquad 1 = \theta \Pi_t^{\epsilon-1} + (1-\theta)\,\widetilde{p}_t^{1-\epsilon}$$

$$R_t = r_t E_t \Pi_{t+1} \qquad \widetilde{p}_t \cdot s_{1,t} = \frac{\epsilon}{\epsilon - 1} \cdot s_{2,t} \qquad div_t = y_t - w_t n_t \qquad p_t^* y_t = a_t n_t$$

$$w_t = -\frac{\frac{\partial U(c_t, n_t^s, z_t)}{\partial n_t^s}}{\frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t}}$$

$$s_{1,t} = y_t \frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t} + \beta\theta E_t \Pi_{t+1}^{\epsilon-1} s_{1,t+1} \qquad p_t^* = (1-\theta)\,\widetilde{p}_t^{-\epsilon} + \theta \Pi_t^{\epsilon} p_{t-1}^*$$

$$s_{2,t} = mc_t y_t \frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t} + \beta\theta E_t \Pi_{t+1}^{\epsilon} s_{2,t+1}$$

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t}$$

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}$$

$$\frac{\partial U(c_t, n_t^s, z_t)}{\partial c_t} = \beta E_t \left[\frac{\partial U(c_{t+1}, n_{t+1}^s, z_{t+1})}{\partial c_{t+1}} r_t \right] \qquad R_t = R \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{y_t}{y} \right)^{\phi_y} e^{\nu_t} \qquad \nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t}$$