Steady State

Total Factor productivity

$$\log(\bar{A}) = \rho_A \cdot \log(\bar{A})$$

$$\Leftrightarrow \bar{A} = 1$$

Marginal Cost:

$$\overline{MC} = 1$$

Euler Equation:

$$\bar{U}^c = \beta \bar{U}^c (1 - \delta + \bar{R})$$

$$\Leftrightarrow \bar{R} = \frac{1}{\beta} + \delta - 1$$

Captial demand:

$$\bar{R} = \overline{MC} \cdot \alpha \cdot \bar{A} \cdot \bar{K}^{\alpha - 1} \cdot \bar{L}^{1-\alpha}$$

$$\Leftrightarrow \frac{\bar{K}}{\bar{L}} = \left(\frac{\overline{MC} \cdot \alpha \cdot \bar{A}}{\bar{R}} \right)^{\frac{1}{1-\alpha}}$$

Labor demand

$$\bar{w} = \overline{MC}\,(1-\alpha)\cdot\bar{A}\cdot\bar{K}^{\alpha}\cdot\bar{L}^{-\alpha} = \overline{MC}(1-\alpha)\cdot\bar{A}\cdot\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}$$

Captital accumulation:

$$\bar{K} = (1-\delta)\cdot\bar{K} + \bar{I}$$

$$\Leftrightarrow \quad \frac{\bar{I}}{\bar{L}} = \delta\cdot\frac{\bar{K}}{\bar{L}}$$

Production function:

$$\bar{Y} = \bar{A}\cdot\bar{K}^{\alpha}\cdot\bar{L}^{1-\alpha} = \bar{A}\cdot\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}\cdot\bar{L}$$

$$\Leftrightarrow \quad \frac{\bar{Y}}{\bar{L}} = \bar{A}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}$$

Market Clearing

$$\frac{\bar{C}}{\bar{L}} = \frac{\bar{Y}}{\bar{L}} - \frac{\bar{I}}{\bar{L}}$$

Deriving Steady-State Labor:

Log-Utility:

Labor supply:

$$\psi \cdot \frac{1}{1-\bar{L}} = \gamma \cdot (\bar{C})^{-1} \cdot W$$

$$\Leftrightarrow \psi \cdot \frac{\bar{L}}{1-\bar{L}} = \gamma \cdot \left(\frac{\bar{C}}{\bar{L}}\right)^{-1} \cdot W$$

$$\Leftrightarrow \bar{L} = (1-\bar{L}) \cdot \frac{\gamma}{\psi} \left(\frac{\bar{C}}{\bar{L}}\right)^{-1} \cdot W$$

$$\Leftrightarrow \bar{L} = \frac{\frac{\gamma}{\psi}\left(\frac{\bar{C}}{\bar{L}}\right)^{-1} \cdot W}{1 + \frac{\gamma}{\psi}\left(\frac{\bar{C}}{\bar{L}}\right)^{-1} \cdot W}$$

CES-Utility

Labor supply:

$$W \cdot \gamma \cdot C^{-\eta_c} = \psi (1-L)^{-\eta_c}$$

$$W \cdot \gamma \cdot \left(\frac{C}{L}\right)^{-\eta_c} = \frac{\psi}{\gamma} (1-L)^{-\eta_c} \cdot L^{\eta_c}$$

Once $\bar{L}$ is computed:

$$\bar{C} = \left(\frac{\bar{C}}{\bar{L}}\right) \cdot \bar{L} \quad , \quad \bar{I} = \left(\frac{\bar{I}}{\bar{L}}\right) \cdot \bar{L} \quad , \quad \bar{Y} = \left(\frac{\bar{Y}}{\bar{L}}\right) \cdot \bar{L}$$

$$\bar{K} = \left(\frac{\bar{K}}{\bar{L}}\right) \cdot \bar{L}$$