RBC Model

Model description

Model structure

Household

Representative household maximizes present as well as expected future utility

$$\max_{\{C_t, I_t, L_t, K_t\}} E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}$$

Functional forms

$$U_t = \gamma \frac{C_t^{1-\eta_C} - 1}{1 - \eta_C} + \psi \frac{(1 - L_t)^{1-\eta_L} - 1}{1 - \eta_L} \quad \text{[CES-utility]}$$

$$U_t = \gamma \log(C_t) + \psi \log(1 - L_t) \quad \text{[log-utility]}$$

Household

Budget constraint (in real terms)

$$C_t + I_t \leq W_t L_t + R_t K_t + \Pi_t$$

Capital accumulation

Law of motion for (end-of-period) capital K_t

$$K_t = (1 - \delta)K_{t-1} + I_t$$

Firms

Production function: $f(K_{t-1}, L_t) = Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$

Profits of representative firm (in real terms): $\Pi_t = Y_t - W_t L_t - R_t K_{t-1}$

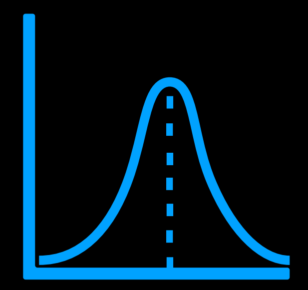
Maximization of expected profits: $\max_{\{L_t, K_{t-1}\}} E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \Pi_{t+j}$

Stochastic discount factor: $Q_{t+j} = \dfrac{\partial U_{t+j} / \partial C_{t+j}}{\partial U_t / \partial C_t}$

Stochastic processes

- Law of motion for productivity

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$$



Model closure

- Non-negativity:

$$K_t, C_t, I_t \geq 0 \text{ and } 0 \leq L_t \leq 1$$

- Clearing of labor and goods market

$$Y_t = C_t + I_t$$

- Transversality condition

$$\lim_{s \to \infty} \beta^s E_t U^C_{t+s} K_{t+s} = 0$$

Household

Optimality conditions

Household: Lagrangian

$$L = E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}\left(C_{t+j}, L_{t+j}\right)$$
$$+\beta^j \lambda_{t+j}\left[W_{t+j}L_{t+j} + R_{t+j}K_{t-1+j} - C_{t+j} - I_{t+j}\right]$$
$$+\beta^j \mu_{t+j}\left[(1-\delta)K_{t-1+j} + I_{t+j} - K_{t+j}\right]$$

Utility function

Budget constraint

Capital accumulation

[j=0] $= U_t\left(C_t, L_t\right) + \lambda_t\left[W_tL_t + R_tK_{t-1} - C_t - I_t\right] + \mu_t\left[(1-\delta)K_{t-1} + I_t - K_t\right]$

[j=1] $+E_t\beta U_{t+1}\left(C_{t+1}, L_{t+1}\right) + E_t\beta\lambda_{t+1}\left[W_{t+1}L_{t+1} + R_{t+1}K_t - C_{t+1} - I_{t+1}\right] + E_t\beta\mu_{t+1}\left[(1-\delta)K_t + I_{t+1} - K_{t+1}\right]$

[j>1] $+E_t\sum_{j=2}^{\infty}\beta^j U_{t+j}\left(C_{t+j}, L_{t+j}\right) + \beta^j\lambda_{t+j}\left[W_{t+j}L_{t+j} + R_{t+j}K_{t-1+j} - C_{t+j} - I_{t+j}\right] + \beta^j\mu_{t+j}\left[(1-\delta)K_{t-1+j} + I_{t+j} - K_{t+j}\right]$

Take derivatives wrt C_t, L_t, I_t, K_t and rearrange to get FOC

Household: FOC

(I) Intertemporal optimality: $U_t^C = \beta E_t \left[U_{t+1}^C \left(1 - \delta + R_{t+1} \right) \right]$

(II) Intratemporal optimality: $W_t = -\dfrac{U_t^L}{U_t^C}$

where $U_t^C = \partial U_t / \partial C_t$ and $U_t^L = \partial U_t / \partial L_t$

- CES-utility: $U_t^C = \gamma C_t^{-\eta_C}$ and $U_t^L = -\psi(1 - L_t)^{-\eta_L}$

- Log-utility: $U_t^C = \gamma C_t^{-1}$ and $U_t^L = -\psi(1 - L_t)^{-1}$

Firm

Optimality conditions

Firm: Lagrangian

$$L = E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \left[Y_{t+j} - W_{t+j} L_{t+j} - R_{t+j} K_{t-1+j} \right]$$
$$+ \beta^j Q_{t+j} MC_{t+j} \left[A_{t+j} K_{t-1+j}^{\alpha} L_{t+j}^{1-\alpha} - Y_t \right]$$

Profits

Production function

[j=0] $= Y_t - W_t L_t - R_t K_{t-1} + MC_t \left[A_t K_{t-1}^{\alpha} L_t^{1-\alpha} - Y_t \right]$

[j=1] $+ E_t \beta Q_{t+1} \left[Y_{t+1} - W_{t+1} L_{t+1} - R_{t+1} K_t \right] + E_t \beta Q_{t+1} MC_{t+1} \left[A_{t+1} K_t^{\alpha} L_{t+1}^{1-\alpha} - Y_{t+1} \right]$

[j>1] $+ E_t \sum_{j=2}^{\infty} \beta^j Q_{t+j} \left[Y_{t+j} - W_{t+j} L_{t+j} - R_{t+j} K_{t-1+j} \right] + \beta^j Q_{t+j} MC_{t+j} \left[A_{t+j} K_{t-1+j}^{\alpha} L_{t+j}^{1-\alpha} - Y_t \right]$

Firm: Lagrangian

Future dynamics don't matter for period t decisions, i.e. Lagrangian reduces to static optimization of

$$L = Y_t - W_t L_t - R_t K_{t-1} + MC_t \left[A_t K_{t-1}^{\alpha} L_t^{1-\alpha} - Y_t \right]$$

Take derivatives wrt Y_t, L_t, K_{t-1} and rearrange to get FOC

Firm: FOC

(III) Marginal costs: $MC_t = 1$

(IV) Labor demand: $W_t = MC_t f_L$

(V) Capital demand: $R_t = MC_t f_K$

where $f_L = \partial f / \partial L_t = (1 - \alpha)\dfrac{Y_t}{L_t}$ and $f_K = \partial f / \partial K_{t-1} = \alpha \dfrac{Y_t}{K_{t-1}}$

Nonlinear Model Equations

Nonlinear model equations

1. $U_t^C = \beta E_t \left[U_{t+1}^C \left(1 - \delta + R_{t+1} \right) \right]$

2. $W_t = - \frac{U_t^L}{U_t^C}$

3. $K_t = (1 - \delta) K_{t-1} + I_t$

4. $Y_t = C_t + I_t$

5. $Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$

6. $MC_t = 1$

7. $W_t = MC_t f_L$

8. $R_t = MC_t f_K$

9. $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

9 Model equations

9 Endogenous variables: $Y_t, C_t, K_t, L_t, A_t, R_t, W_t, I_t, MC_t$

1 Exogenous variable: ε_t^A