Household

$$
\begin{aligned}
\mathcal{L} = {} & U_t(C_t, L_t) + \lambda_t \left(W_t \cdot L_t + R_t \cdot K_{t-1} - C_t - I_t \right) \\
& + \mu_t \left((1-\delta) K_{t-1} + I_t - K_t \right) \\
& + E_t \beta\, U_{t+1}(C_{t+1}, L_{t+1}) + E_t \beta \lambda_{t+1} \left(W_{t+1} \cdot L_{t+1} + R_{t+1} \cdot K_t - C_{t+1} - I_{t+1} \right) \\
& + E_t \beta \mu_{t+1} \left((1-\delta) K_t + I_{t+1} - K_{t+1} \right) \\
& + E_t \sum_{j=2}^{\infty} \beta^j U_{t+j}(C_{t+j}, L_{t+j}) + \beta^j \lambda_{t+j} \left(W_{t+j} \cdot L_{t+j} + R_{t+j} \cdot K_{t+j-1} - C_{t+j} - I_{t+j} \right.
\end{aligned}
$$

Derivative wrt C_t

$$\frac{\partial \mathcal{L}}{\partial C_t} = U_t^C - \lambda_t = 0$$

$$\Leftrightarrow \quad \lambda_t = U_t^C \qquad (1)$$

Derivative wrt L_t

$$\frac{\partial \mathcal{L}}{\partial L_t} = U_t^L + \lambda_t \cdot W_t = 0$$

$$\Leftrightarrow \quad \lambda_t \cdot W_t = -U_t^L \qquad (2)$$

Derivative wrt I_t

$$\frac{\partial L}{\partial I_t} = -\lambda_t + \mu_t = 0$$

$$\Leftrightarrow \quad \mu_t = \lambda_t \qquad (3)$$

Derivative wrt K_t:

$$\frac{\partial L}{\partial K_t} = -\mu_t + E_t \beta\left(\lambda_{t+1} \cdot R_{t+1} + \mu_{t+1}(1-\delta)\right) = 0$$

$$\Leftrightarrow \quad \mu_t = E_t \beta\left(\mu_{t+1}(1-\delta) + \lambda_{t+1} \cdot R_{t+1}\right)$$

$$(4)$$

(1) and (3) in (4)

$$U_t^c = \beta \cdot E_t \, U_{t+1}^c \left(1 - \delta + R_{t+1}\right) \qquad \mathrm{I}$$

(1) in (2)

$$W_t = \frac{-U_t^L}{U_t^c} \qquad \mathrm{II}$$

Firm

$$\mathcal{L} = Y_t - W_t \cdot L_t - R_t \cdot K_{t-1} + MC_t \left(A_t \cdot K_{t-1}^{\alpha} \cdot L_t^{1-\alpha} - Y_t \right)$$

$$\frac{\partial \mathcal{L}}{\partial Y_t} = 1 - MC_t = 0$$

$$\Leftrightarrow \quad MC_t = 1 \qquad \text{III}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -W_t + MC_t \cdot A_t \cdot K_t^{\alpha} \cdot L_t^{-\alpha} \cdot (1-\alpha) = 0$$

$$\Leftrightarrow \quad W_t = MC_t (1-\alpha) \cdot A_t \cdot K_t^{\alpha} \cdot L_t^{-\alpha} \cdot \frac{L_t}{L_t}$$

$$= MC_t (1-\alpha) \cdot \frac{Y_t}{L_t} \qquad \text{IV}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t-1}} = -R_t + MC_t \cdot A_t \cdot K_t^{\alpha-1} \cdot \alpha \cdot L_t^{1-\alpha} = 0$$

$$\Leftrightarrow \quad R_t = MC_t \alpha \cdot A_t \cdot K_t^{\alpha-1} \cdot L_t^{1-\alpha} \cdot \frac{K_t}{K_t}$$

$$= MC_t \cdot \alpha \cdot \frac{Y_t}{K_t} \qquad \text{V}$$